Question	Scheme	Marks
7(a)	$\frac{\mathrm{d}x}{\mathrm{d}t} = k - 3x \Rightarrow \int \frac{1}{k-3x}\,\mathrm{d}x = \int \mathrm{d}t$ oe	M1
	$\Rightarrow -\frac{1}{3}\ln\left(k-3x\right) = t\left(+c\right)$	M1 A1
	$t = 0, x = 0 \Rightarrow -\frac{1}{3}\ln k = c$	M1
	$-\ln\left(k-3x\right) = 3t - \ln k \Rightarrow \ln\frac{k-3x}{k} = -3t \Rightarrow 1 - \frac{3}{k}x = \mathrm{e}^{-3t} \Rightarrow x = ...$	M1
	$x = \frac{k}{3}\left(1-\mathrm{e}^{-3t}\right)$	A1
		(6)
(b)	$t \to \infty \Rightarrow \mathrm{e}^{-3t} \Rightarrow 0 \Rightarrow x \to \frac{k}{3} \Rightarrow \frac{k}{3} = 7 \Rightarrow k = ...$	M1
	$k = 21$	A1
		(2)
(c)	$x = 5 \Rightarrow 7\left(1-\mathrm{e}^{-3t}\right) = 5 \Rightarrow \mathrm{e}^{-3t} = ...$	M1
	$\Rightarrow t = -\frac{1}{3}\ln\frac{2}{7}$	dM1
	Awrt 0.42 seconds	A1
		(3)
		(11 marks)

Alt (a): Use the same criteria for the first 3 marks. See notes

7(a)	$\frac{\mathrm{d}x}{\mathrm{d}t} = k - 3x \Rightarrow \int_0^x \frac{1}{k-3x}\,\mathrm{d}x = \int_0^t \mathrm{d}t$ oe	M1
	$\Rightarrow \left[-\frac{1}{3}\ln\left(k-3x\right)\right]_0^x = \left[t\right]_0^t$	M1 A1
	$-\frac{1}{3}\ln\left(k-3x\right) + \frac{1}{3}\ln k = t$	M1
	$\ln k - \ln\left(k-3x\right) = 3t \Rightarrow \ln\frac{k}{k-3x} = 3t \Rightarrow \frac{k}{k-3x} = \mathrm{e}^{3t} \Rightarrow k = k\mathrm{e}^{3t} - 3x\mathrm{e}^{3t} \Rightarrow x = ...$	M1
	$x = \frac{k}{3}\left(1-\mathrm{e}^{-3t}\right)$	A1

Mark as one complete question. For example, it is possible to get (b) without doing (a)

(a)

M1: Separates the variables correctly, with or without the integration signs shown. The dx and dt must be shown in the correct positions

M1: Attempts the integration of the LHS with $\frac{1}{k-3x} \to \pm a\ln\left(k-3x\right)$ A missing bracket should be penalised unless recovered later in the solution

A1: Fully correct integration, with or without the + *c* for this mark.

A missing bracket should be penalised unless recovered later in the solution

M1: Attempts to find the constant of integration following **any** attempt at integration to an equation involving a constant. This involves $t = 0, x = 0 \Rightarrow c = ...$ where *c* is non zero. For a definite integral we would require the $x = 0$ to be clearly substituted to obtain a constant term.

M1: Correct log work to make *x* the subject. May be scored before the previous M if the constant is found later. Allow if the constant is zero, or is missing. Condone slips on signs.

You may see versions of the following showing correct exponential work:

$$-\frac{1}{3}\ln\left(k-3x\right)=t+c \Rightarrow \ln\left(k-3x\right)=-3t+d \Rightarrow k-3x=\mathrm{e}^{-3t+d} \Rightarrow k-3x=A\mathrm{e}^{-3t} \Rightarrow x=\frac{k-A\mathrm{e}^{-3t}}{3}$$

A1: Correct answer or exact equivalent. E.g. $x=\frac{k\left(\mathrm{e}^{3t}-1\right)}{3\mathrm{e}^{3t}}$.

(b)

M1: Correctly identifies $\mathrm{e}^{-3t} \to 0$ (which may be implied) and deduces the long-term limit for their expression and sets their long-term limit to 7 and solves for *k*. The equation must be of the form $x = p - q\mathrm{e}^{-3t}$ with *p* in terms of *k* and *q* > 0. Alternatively, if they have an equation of the form $x=\frac{k\left(\mathrm{e}^{3t}-\beta\right)}{\delta\mathrm{e}^{3t}}$ they would need to solve $7=\frac{k}{\delta}$ to find the value of *k.*

Condone work on equations of the same form where they have lost the 3 and used another positive value, usually 1. So, score for equations $x = p - q\mathrm{e}^{-\omega t}$ or $x=\frac{k\left(\mathrm{e}^{\omega t}-\beta\right)}{\delta\mathrm{e}^{\omega t}}\ \omega>0$ with the same conditions.

A1: $k = 21$ following a correct equation.

Correct answer following correct equation with no working scores both marks

Alt (b)

M1: Correctly identifies $\frac{\mathrm{d}x}{\mathrm{d}t}=k-3x=0$ and substitutes in $x = 7$ leading to a value for *k*

A1: $k = 21$

(c)

M1: Substitutes $x = 5$ and their value for *k* into their equation from (a), then rearranges to $\mathrm{e}^{\pm 3t} = ...$

This is dependent upon having a **solvable** equation of the form $x = \pm p \pm q\mathrm{e}^{-3t}$ o.e such as $x=\frac{\pm\mu\mathrm{e}^{3t}\pm\kappa}{\mathrm{e}^{3t}}$

Condone work on equations of the same form where they have lost the 3 and used another positive value, usually 1. So, score for equations $x = \pm p \pm q\mathrm{e}^{-\omega t}$ or $x=\frac{\pm\mu\mathrm{e}^{\omega t}\pm\kappa}{\mathrm{e}^{\omega t}}\ \omega>0$ with the same conditions.

dM1: Solves an equation of the form $\mathrm{e}^{\pm 3t} = k, k > 0$ to find a value for *t.* Condone work in solving equations of the form $\mathrm{e}^{\pm\omega t} = k, k > 0$ following the loss of the 3

A1: cso 0.42 (seconds)

Alt (c)

M1: Substitutes $x = 5$ into their **solvable** equation of the form $\pm\beta\ln\left(k-3x\right)=t\pm\delta\ln k$ with their *k*

dM1: ... And finds a value for *t*

A1: cso awrt 0.42 (seconds)

Question	Scheme	Marks
8(a)	$(8-3x)^{\frac{4}{3}}=16\left(1-\frac{3}{8}x\right)^{\frac{4}{3}}$ but condone $16(1-kx)^{\frac{4}{3}}, k\neq 3$	**M1**
	"Correct" term 3 or term 4 in $(1-kx)^{\frac{4}{3}}=1\pm\frac{4}{3}kx+\frac{\frac{4}{3}\times\frac{1}{3}}{2}(\pm kx)^2+\frac{\frac{4}{3}\times\frac{1}{3}\times\left(-\frac{2}{3}\right)}{6}(\pm kx)^3+...$	**M1**
	Two terms correct of $16-8x+\frac{x^2}{2}+\frac{x^3}{24}+...$ following M1, M1 and $k=\pm\frac{3}{8}$	**A1**
	$=16-8x+\frac{x^2}{2}+\frac{x^3}{24}+...$	**A1**
		(4)
(b)	Assume the curves meet so $8x-\frac{15x^2}{2}+8="16-8x+\frac{x^2}{2}+\frac{x^3}{24}"$ (for some x)	**M1**
	$\Rightarrow 0=8-16x+8x^2+\frac{x^3}{24}+...\Rightarrow 0=8\left(x^2-2x\right)+8+\frac{x^3}{24}+...\Rightarrow 8\left(x-1\right)^2\pm..+..$	**M1**
	$\Rightarrow 0=8\left(x-1\right)^2+\frac{x^3}{24}$	**A1**
	Requires • Argument that $8\left(x-1\right)^2+\frac{x^3}{24}>0$. E.g. $8\left(x-1\right)^2\geqslant 0$ and $\frac{x^3}{24}>0$ (for $0<x<\frac{8}{3}$) so $8\left(x-1\right)^2+\frac{x^3}{24}>0$ • Minimal statement (contradiction) and conclusion	**A1**
		(4)
		(8 marks)

(a)

M1: Attempts to takes out a factor of 16 or $8^{\frac{4}{3}}$. Look for $(8-3x)^{\frac{4}{3}}=16(1-kx)^{\frac{4}{3}},\quad k\neq 3$

or $(8-3x)^{\frac{4}{3}}=8^{\frac{4}{3}}(1-kx)^{\frac{4}{3}},\quad k\neq 3$ Condone incorrect values of k other than 3

M1: Attempts the binomial expansion on $\left(1-kx\right)^{\frac{4}{3}}$ achieves at least a "correct" term 3 or term 4 (which may be unsimplified) for their k. Even allow with $k=1$

Look for either $\frac{\frac{4}{3}\times\frac{1}{3}}{2}(\pm kx)^2$ or $\frac{\frac{4}{3}\times\frac{1}{3}\times\left(-\frac{2}{3}\right)}{6}(\pm kx)^3$ with the correct binomial coefficient and the correct power of x. Condone missing brackets on the $\left(kx\right)$ terms.

If they expand $\left(1-\frac{3}{8}x\right)^{\frac{4}{3}}$ without showing the unsimplified terms it is implied by $\pm\frac{x^3}{384}$

A1: For two correct and simplified terms of $16-8x+\frac{x^2}{2}+\frac{x^3}{24}+...$ following M1, M1 and $k=\pm\frac{3}{8}$

A1: Correct expansion from correct work. Terms may be as a list for both accuracy marks