Question	Scheme	Marks
6(a)	$\left\|\overrightarrow{OA}\right\|^2 = \left(1+8\lambda\right)^2 + \left(2-\lambda\right)^2 + \left(5+4\lambda\right)^2$	**M1**
	$\left\|\overrightarrow{OA}\right\| = 5\sqrt{10} \Rightarrow \left(1+8\lambda\right)^2 + \left(2-\lambda\right)^2 + \left(5+4\lambda\right)^2 = 250$	**M1**
	$\Rightarrow 64\lambda^2 + 16\lambda + 1 + \lambda^2 - 4\lambda + 4 + 16\lambda^2 + 40\lambda + 25 = 250$ $\Rightarrow 81\lambda^2 + 52\lambda - 220 = 0$*	**A1***
		(3)
(b)	$81\lambda^2 + 52\lambda - 220 = 0 \Rightarrow \left(81\lambda - 110\right)\left(\lambda + 2\right) = 0 \Rightarrow \lambda = ... \left(-2, \frac{110}{81}\right)$ $\Rightarrow \overrightarrow{OA} = \begin{pmatrix} 1+8\times"\text{their}\lambda" \\ 2-"\text{their}\lambda" \\ 5+4\times"\text{their}\lambda" \end{pmatrix}$	**M1**
	E.g $\Rightarrow \overrightarrow{OA} = \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix} - 2\begin{pmatrix} 8 \\ -1 \\ 4 \end{pmatrix} = \begin{pmatrix} -15 \\ 4 \\ -3 \end{pmatrix}$* ; or $\overrightarrow{OA} = \begin{pmatrix} 1+8\times\frac{110}{81} \\ 2-\frac{110}{81} \\ 5+4\times\frac{110}{81} \end{pmatrix} = \begin{pmatrix} \frac{961}{81} \\ \frac{52}{81} \\ \frac{845}{81} \end{pmatrix}$	**A1*;** **A1**
		(3)
(c)	$\cos\theta = \pm\frac{\begin{pmatrix} -15 \\ 4 \\ -3 \end{pmatrix}.\begin{pmatrix} 8 \\ -1 \\ 4 \end{pmatrix}}{\sqrt{15^2+4^2+3^2}\sqrt{8^2+1^2+4^2}} = ...$	**M1**
	$= \pm\frac{-136}{45\sqrt{10}}$ $\left(= \pm 0.9557...\right)$ or e.g. $\theta =$ awrt $0.299\,(rad)$ / $17.1°$	**A1**
	Area $OAB = \frac{1}{2}5\sqrt{10}\times 4\sqrt{10}\sin\theta = ...$	**M1**
	= awrt 29.4	**A1**
		(4)
		(10 marks)

(a)

M1: Attempts to find the distance, or distance squared, from *O* to a general point on the line, or to *A*. Condone slips. It is acceptable to use a different variable rather than λ.

M1: Attempts to use $\left|\overrightarrow{OA}\right| = 5\sqrt{10}$ to set up a quadratic equation in λ, need not be expanded for this mark.

Condone slips on the 250 for OA^2 on this mark. Allow it to appear as $5\times 10 = 50$

A1*: Expands and reaches the correct equation from correct work.

An intermediate line equivalent to $64\lambda^2 + 16\lambda + 1 + \lambda^2 - 4\lambda + 4 + 16\lambda^2 + 40\lambda + 25 = 250$ must be written out before the final given answer is seen.

(b)

M1: Correct attempt at solving the quadratic equation, (allow a calculator), and substitutes at least one of their values for λ into the equation for the line to find a possible position vector. The $(-15, 4, -3)$ is given so they cannot just write $\lambda = -2$, $A = (-15, 4, -3)$ without some evidence. However, sight of a correct second coordinate $\left(\frac{961}{81}, \frac{52}{81}, \frac{845}{81}\right)$ would imply M1

A1*: Correct value $\lambda = -2$ found, and substituted to reach the given position vector.

Scored for sight of $\overrightarrow{OA} = \begin{pmatrix} 1+8\times-2 \\ 2-(-2) \\ 5+4\times-2 \end{pmatrix} = \begin{pmatrix} -15 \\ 4 \\ -3 \end{pmatrix}$ o.e.

Also allow the coordinate $(-15, 4, -3)$ but not an incorrect hybrid version. E.g. $(-15\mathbf{i}, 4\mathbf{j}, -3\mathbf{k})$

A1: Correct second possible position vector but condone in coordinate form. $\left(\frac{961}{81}, \frac{52}{81}, \frac{845}{81}\right)$ Must be exact. Do not allow hybrid versions but if this form has been penalised already, allow here.

(c)

M1: Attempts the scalar product of $\pm\overrightarrow{OA} = \pm(-15\mathbf{i} + 4\mathbf{j} - 3\mathbf{k})$ and the direction vector of the line $\pm(8\mathbf{i} - 1\mathbf{j} + 4\mathbf{k})$ to find $\cos\theta$ for the angle between $\overrightarrow{OA}$ and l_2. Condone slips but the intention must be clear. If the method of finding modulus is not shown award if one of the two is correct. For example, $4\sqrt{10}$ is a common error/slip on the modulus of $\pm(8\mathbf{i} - 1\mathbf{j} + 4\mathbf{k})$ as this length is given in the question.

Note that it is acceptable to find the scalar product of $\pm\overrightarrow{OA}$ with $\pm(8\mu\mathbf{i} - 1\mu\mathbf{j} + 4\mu\mathbf{k})$ as long as μ is used consistently within the calculation on the numerator. Note any variable may be used here.

A1: Correct value for $\cos\theta$ (either sign acceptable). Implied by use of $\theta =$ awrt 0.299 rads or awrt 17.1°

M1: Uses formula for area of triangle in a complete method to find the area required. It is dependent upon having found $\cos\theta$ via a scalar product of the two correct vectors.

A1: Correct area, awrt 29.4

Alt: There are alternative methods most of which involve finding the perpendicular distance between the two lines. An example of such a method is shown below.

Alt (c)	Midpoint of possible A's is $M = \frac{1}{81}\begin{pmatrix} -127 \\ 188 \\ 301 \end{pmatrix}$ and $OM = \frac{1}{81}\sqrt{127^2 + 188^2 + 301^2} = ...$	**M1**
	$= 4.6534\ldots$	**A1**
	Area $= \frac{1}{2} \times 4\sqrt{10} \times 4.6534... ; = 29.43\ldots$	**M1; A1**
		(4)

M1: Finds the midpoint for the two possible positions for A and proceeds to find the distance from O to this point.

A1: Correct distance between lines.

M1: Uses formula for area of triangle in a complete method to find the area required.

A1: Correct area, awrt 29.4

There may be longer alternatives, such as finding point B first and using a different angle, but the scheme will apply in like manner.

Question	Scheme	Marks
7(a)	$\frac{\mathrm{d}x}{\mathrm{d}t} = k - 3x \Rightarrow \int \frac{1}{k-3x}\,\mathrm{d}x = \int \mathrm{d}t$ oe	M1
	$\Rightarrow -\frac{1}{3}\ln\left(k-3x\right) = t\left(+c\right)$	M1 A1
	$t = 0, x = 0 \Rightarrow -\frac{1}{3}\ln k = c$	M1
	$-\ln\left(k-3x\right) = 3t - \ln k \Rightarrow \ln\frac{k-3x}{k} = -3t \Rightarrow 1 - \frac{3}{k}x = \mathrm{e}^{-3t} \Rightarrow x = ...$	M1
	$x = \frac{k}{3}\left(1-\mathrm{e}^{-3t}\right)$	A1
		(6)
(b)	$t \to \infty \Rightarrow \mathrm{e}^{-3t} \Rightarrow 0 \Rightarrow x \to \frac{k}{3} \Rightarrow \frac{k}{3} = 7 \Rightarrow k = ...$	M1
	$k = 21$	A1
		(2)
(c)	$x = 5 \Rightarrow 7\left(1-\mathrm{e}^{-3t}\right) = 5 \Rightarrow \mathrm{e}^{-3t} = ...$	M1
	$\Rightarrow t = -\frac{1}{3}\ln\frac{2}{7}$	dM1
	Awrt 0.42 seconds	A1
		(3)
		(11 marks)

Alt (a): Use the same criteria for the first 3 marks. See notes

7(a)	$\frac{\mathrm{d}x}{\mathrm{d}t} = k - 3x \Rightarrow \int_0^x \frac{1}{k-3x}\,\mathrm{d}x = \int_0^t \mathrm{d}t$ oe	M1
	$\Rightarrow \left[-\frac{1}{3}\ln\left(k-3x\right)\right]_0^x = \left[t\right]_0^t$	M1 A1
	$-\frac{1}{3}\ln\left(k-3x\right) + \frac{1}{3}\ln k = t$	M1
	$\ln k - \ln\left(k-3x\right) = 3t \Rightarrow \ln\frac{k}{k-3x} = 3t \Rightarrow \frac{k}{k-3x} = \mathrm{e}^{3t} \Rightarrow k = k\mathrm{e}^{3t} - 3x\mathrm{e}^{3t} \Rightarrow x = ...$	M1
	$x = \frac{k}{3}\left(1-\mathrm{e}^{-3t}\right)$	A1

Mark as one complete question. For example, it is possible to get (b) without doing (a)

(a)

M1: Separates the variables correctly, with or without the integration signs shown. The dx and dt must be shown in the correct positions

M1: Attempts the integration of the LHS with $\frac{1}{k-3x} \to \pm a\ln\left(k-3x\right)$ A missing bracket should be penalised unless recovered later in the solution