Question	Scheme	Marks
6(a)	$\left(3\mathbf{i}+p\mathbf{j}+7\mathbf{k}\right)+\lambda\left(2\mathbf{i}-5\mathbf{j}+4\mathbf{k}\right)=\left(8\mathbf{i}-2\mathbf{j}+5\mathbf{k}\right)+\mu\left(4\mathbf{i}+\mathbf{j}+2\mathbf{k}\right)$ $3+2\lambda=8+4\mu$ (1) $\Rightarrow p-5\lambda=-2+\mu$ (2) $7+4\lambda=5+2\mu$ (3)	**M1**
	e.g. $(3)-2\times(1):1=-11-6\mu\Rightarrow\mu=-2$ $\left(\text{or } \lambda=-\frac{3}{2}\right)$	**M1**
	$\mu=-2$, $\lambda=-\frac{3}{2}\Rightarrow p=-2-2-\frac{15}{2}=...$	**M1**
	$p=-\frac{23}{2}$	**A1**
		(4)
(b)	Intersect at $\left(8\mathbf{i}-2\mathbf{j}+5\mathbf{k}\right)+-2\left(4\mathbf{i}+\mathbf{j}+2\mathbf{k}\right)=..$	**M1**
	$=-4\mathbf{j}+\mathbf{k}$	**A1**
		(2)
(c)	$\left(2\mathbf{i}-5\mathbf{j}+4\mathbf{k}\right).\left(4\mathbf{i}+\mathbf{j}+2\mathbf{k}\right)=2\times4-5\times1+4\times2=...$	**M1**
	$\cos\theta=\frac{"11"}{\sqrt{2^2+5^2+4^2}\sqrt{4^2+1^2+2^2}}=...\left(=\frac{11}{3\sqrt{105}}=0.3578...\right)$	**M1**
	$\theta=\cos^{-1}\frac{11}{3\sqrt{105}}=69.0°$	**A1**
		(3)
(d)	$\lambda=2\Rightarrow\overrightarrow{OA}=\left(7\mathbf{i}-\frac{43}{2}\mathbf{j}+15\mathbf{k}\right)$	**B1ft**
	$\overrightarrow{AB}=\pm\left(\left(8\mathbf{i}-2\mathbf{j}+5\mathbf{k}\right)+\mu\left(4\mathbf{i}+\mathbf{j}+2\mathbf{k}\right)-\left(7\mathbf{i}-\frac{43}{2}\mathbf{j}+15\mathbf{k}\right)\right)$ $=\pm\left(\left(1+4\mu\right)\mathbf{i}+\left(\frac{39}{2}+\mu\right)\mathbf{j}+\left(-10+2\mu\right)\mathbf{k}\right)$	**M1**
	$\left(\left(1+4\mu\right)\mathbf{i}+\left(\frac{39}{2}+\mu\right)\mathbf{j}+\left(-10+2\mu\right)\mathbf{k}\right).\left(4\mathbf{i}+\mathbf{j}+2\mathbf{k}\right)=0\Rightarrow\mu=..\left(=-\frac{1}{6}\right)$	**M1**
	$\overrightarrow{OB}=\left(8\mathbf{i}-2\mathbf{j}+5\mathbf{k}\right)+"-\frac{1}{6}"\left(4\mathbf{i}+\mathbf{j}+2\mathbf{k}\right)=...$	**dM1**
	$=\frac{22}{3}\mathbf{i}-\frac{13}{6}\mathbf{j}+\frac{14}{3}\mathbf{k}$ or $\left(\frac{22}{3},-\frac{13}{6},\frac{14}{3}\right)$	**A1**
		(5)
		(14 marks)

Notes:

Accept alternative notations, such as column or row vector notation, throughout. Condone if **i**, **j** and **k** are left in columns etc. except for in the final answers to (b) and (d).

(a) Note: work for (a) must be shown in part (a) not recovered from later parts.

M1: Equates equations of the lines and extracts at least two simultaneous equations (usually (1) and (3)) by equating coefficients.

M1: Solves the equations to find at least one of λ or μ

M1: Uses their values or other suitable method to find *p* allowing for slips.

A1: Correct value.

(b)

M1: Uses either parameter to find the point of intersection. May be implied by two correct entries if no method shown.

A1: Correct point. Accept as coordinates or in vector form.

(c)

M1: Attempts the scalar product with the two direction vectors (or any multiples of them). Allow if there are miscopies etc if the intent of the two correct vectors is clear. Note that $-3\mathbf{i}+\frac{15}{2}\mathbf{j}-6\mathbf{k}$ and $-8\mathbf{i}-2\mathbf{j}-4\mathbf{k}$ are commonly seen correct direction vectors.

M1: Applies the scalar product formula to find the value for cos θ. Allow if wrong "direction" vectors are used, or if a slip is made but the intent is clear. This mark is for demonstrating a correct use of the formula with any vectors.

A1: Correct angle, awrt 69.0°, and isw if they give go on to give the obtuse angle. But A0 if another incorrect angle is found.

(d)

B1ft: Correct coordinates for *A*, follow through their *p*

M1: Attempts to find $\overrightarrow{AB}$ in terms of μ with their $\overrightarrow{OA}$ and a general point on l_2. Allow subtraction either way round. May be implied by two correct entries if no method shown. Note, if $\overrightarrow{AB}=(x-7)\mathbf{i}+\left(y+"\frac{43}{2}"\right)\mathbf{j}+(z-15)\mathbf{k}$ is used this mark is not scored until the point where they identify *x*, *y* and *z* in terms of μ.

M1: Takes scalar product of their $\overrightarrow{AB}$ with direction of l_2, sets equal to zero and solves to find μ

dM1: Depends on previous M mark. Uses their parameter in the correct line to find the coordinates. Implied by 2 correct coordinates if no method is shown.

A1: Correct answer. Accept as coordinates or in vector form.

Alt (d)	$\lambda = 2 \Rightarrow \overrightarrow{OA} = \left(7\mathbf{i} - \frac{43}{2}\mathbf{j} + 15\mathbf{k}\right)$	**B1ft**
E.g.	$\overrightarrow{XA} = \pm\left(\left(7\mathbf{i} - \frac{43}{2}\mathbf{j} + 15\mathbf{k}\right) - \left(-4\mathbf{j} + \mathbf{k}\right)\right)$ $\overrightarrow{XB} = \pm\left(\left(8+4\mu\right)\mathbf{i} + \left(-2+\mu\right)\mathbf{j} + \left(5+2\mu\right)\mathbf{k} - \left(-4\mathbf{j}+\mathbf{k}\right)\right)$	**M1**
	$AX\cos\theta = BX \Rightarrow \left(7^2 + \left(-\frac{35}{2}\right)^2 + 14^2\right)\left(\frac{121}{945}\right) = \left(8+4\mu\right)^2 + \left(2+\mu\right)^2 + \left(4+2\mu\right)^2$ $\Rightarrow 36\mu^2 + 144\mu + 23 = 0 \Rightarrow \mu = ..\left(= -\frac{1}{6}, -\frac{23}{6}\right)$	**M1**
	$\overrightarrow{OB} = \left(8\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}\right) + "-\frac{1}{6}"\left(4\mathbf{i} + \mathbf{j} + 2\mathbf{k}\right) = ...$	**dM1**
	$= \frac{22}{3}\mathbf{i} - \frac{13}{6}\mathbf{j} + \frac{14}{3}\mathbf{k}$ or $\left(\frac{22}{3}, -\frac{13}{6}, \frac{14}{3}\right)$	**A1**
		(5)

Notes

Alt (d) – by right triangle

B1ft: Correct coordinates for *A*, follow through their *p*

M1: Attempts to find at least two sides of the triangle *AXB* where *X* is the intersection point. (Note if no further progress in this method is made the main scheme can apply to allow M1 for just $\overrightarrow{AB}$) in terms of μ where relevant, using their $\overrightarrow{OA}$ and a general point on l_2. Allow subtraction either way round. May be implied by two correct entries if no method shown.

M1: For a full, correct method leading to a value for μ. There are variation to that shown in the scheme, such as use of Pythagoras on all three sides, or via $AX\sin\theta = AB$. The correct side must be used for the hypotenuse, but allow use of decimals in the trig work but must be working in the correct mode to 3s.f..

dM1: Depends on previous M mark. Uses one of their parameters in the correct line to find the coordinates. Implied by 2 correct coordinates if no method is shown. They may use either solution to their quadratic for this mark.

A1: Correct answer only. Must have selected the root which corresponds to *A* and reject the other. Accept as coordinates or in vector form. Allow awrt 3 s.f. for the coordinates.

Question	Scheme	Marks
7(a)	$u = 4x + 2\sin 2x \Rightarrow \dfrac{\mathrm{d}u}{\mathrm{d}x} = 4 + 4\cos 2x$ oe	**M1 A1**
	$\dfrac{\mathrm{d}u}{\mathrm{d}x} = 4 + 4\cos 2x = 8\cos^2 x \Rightarrow \int_{(0)}^{\left(\frac{\pi}{2}\right)} \mathrm{e}^{4x+2\sin 2x}\cos^2 x\,\mathrm{d}x = \int_{(0)}^{(2\pi)} \dfrac{\mathrm{e}^u}{8}\,\mathrm{d}u$	**M1**
	$= \left[\text{"}\dfrac{1}{8}\text{"}\mathrm{e}^u\right]_0^{2\pi}$ or $\left[\text{"}\dfrac{1}{8}\text{"}\mathrm{e}^{4x+2\sin 2x}\right]_0^{\frac{\pi}{2}}$	**A1ft**
Alt by inspection for first 4 marks	$\int \mathrm{e}^{4x+2\sin 2x}\cos^2 x\,\mathrm{d}x = \int \mathrm{e}^{4x+2\sin 2x} \times \dfrac{1}{2}\left(1 \pm \cos 2x\right)\mathrm{d}x$	**(M1 A1)**
	$= k\mathrm{e}^{4x+2\sin 2x}$; $= \left[\text{"}\dfrac{1}{8}\text{"}\mathrm{e}^{4x+2\sin 2x}\right]_0^{\frac{\pi}{2}}$	**(M1; A1ft)**
	$= \left[\dfrac{1}{8}\mathrm{e}^u\right]_0^{2\pi}$ or $\left[\dfrac{1}{8}\mathrm{e}^{4x+2\sin 2x}\right]_0^{\frac{\pi}{2}} = \dfrac{1}{8}\left(\mathrm{e}^{2\pi} - 1\right)$*	**A1*cso**
		(5)
(b)	$V\left[= \pi\int_0^{\frac{\pi}{2}} y^2\,\mathrm{d}x = \pi\int_0^{\frac{\pi}{2}} \left(6\mathrm{e}^{2x+\sin 2x}\cos x\right)^2\,\mathrm{d}x\right] = 36\pi\int_0^{\frac{\pi}{2}} \mathrm{e}^{4x+2\sin 2x}\cos^2 x\,\mathrm{d}x$	**B1**
	$K\int_0^{\frac{\pi}{2}} \mathrm{e}^{4x+2\sin 2x}\cos^2 x\,\mathrm{d}x = \dfrac{K}{8}\left(\mathrm{e}^{2\pi} - 1\right)$	**M1**
	$= \dfrac{9\pi}{2}\left(\mathrm{e}^{2\pi} - 1\right)$	**A1**
		(3)
		(8 marks)

Notes:

(a)

M1: Attempts the derivative of *u*. Look for $\sin 2x \to ...\cos 2x$ May apply double angle formula first, look for $u = 4x + k\sin x\cos x \to \dfrac{\mathrm{d}u}{\mathrm{d}x} = \left(4 + \right)k\left(\cos^2 x - \sin^2 x\right)$ oe. Form should be correct but there may be sign errors.

A1: Correct derivative (any form).

M1: Applies a double angle formula – must be seen – and carries out the full substitution to get an integral in terms of *u*. Must include an attempt to replace d*x*. Do not be concerned with the limits for this mark.

A1ft: Integrates to achieve the correct form with their constant with correct limits assigned (either in *u* or reverted in *x*)– must have scored the M1.