Question	Scheme	Marks
3(a)	$y^2x+3y=4x^2+k$	
	$y^2x \to ...\times y\dfrac{\mathrm{d}y}{\mathrm{d}x}(+...)$ or $3y \to 3\dfrac{\mathrm{d}y}{\mathrm{d}x}$	**M1**
	$y^2x \to 2xy\dfrac{\mathrm{d}y}{\mathrm{d}x}+y^2$	**A1**
	$...+3y=4x^2+k \to ...+3\dfrac{\mathrm{d}y}{\mathrm{d}x}=8x$	**A1**
	$2xy\dfrac{\mathrm{d}y}{\mathrm{d}x}+y^2+3\dfrac{\mathrm{d}y}{\mathrm{d}x}=8x \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x}=...$	**dM1**
	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{8x-y^2}{2xy+3}$ oe	**A1**
		(5)
(b)	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=0 \Rightarrow 8x-4=0 \Rightarrow x=...$	**M1**
	$p=\dfrac{1}{2}$	**A1**
	$k=2^2\times\dfrac{1}{2}+3\times2-4\times\dfrac{1}{4}=...$	**M1**
	$k=7$	**A1**
		(4)
Alt (b)	$(p,2)$ on $C \Rightarrow 4p+6=4p^2+k \Rightarrow 4p^2-4p+k-6=0$ $\Rightarrow 4\left(p-\dfrac{1}{2}\right)^2-1+k-6=0$ or $b^2-4ac=16-16(k-6)$	**M1**
	(Min, single solution) $\Rightarrow p=\dfrac{1}{2}$ or $16-16(k-6)=0 \Rightarrow k=7$	**A1**
	$\Rightarrow k=7-4\left(\dfrac{1}{2}-\dfrac{1}{2}\right)^2$ or $4p^2-4p+7-6=0 \Rightarrow p=...$	**M1**
	The other of $p=\dfrac{1}{2}$ and $k=7$	**A1**
		(4)
		(9 marks)

Notes:

(a)

M1: Some evidence of an attempt at implicit differentiation. Look for one of the terms being differentiated with either the $3\dfrac{\mathrm{d}y}{\mathrm{d}x}$ or a term with $kxy\dfrac{\mathrm{d}y}{\mathrm{d}x}$ or $y^2 \to ky\dfrac{\mathrm{d}y}{\mathrm{d}x}$ as part of a product rule seen appropriately.

A1: y^2x differentiated correctly

A1: $3y \to 3\dfrac{\mathrm{d}y}{\mathrm{d}x}$ and $4x^2 + k \to 8x$

dM1: Make $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ the subject with correct terms factored (but allow e.g. sign errors rearranging or slips in copying). There must be at least two $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ terms in their expression.

A1: Correct expression, accept any equivalent.

(b) Note: allow all marks in (b) from a correct numerator in (a) if the denominator is incorrect.

M1: Sets the numerator of their $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ to zero and substitutes $y = 2$ and solves for *x*. They must have a *y* term in their numerator to be able to access this mark. If the numerator also contains a *k* they will need to also use the equation of the curve to find a second equation in *p* and *k* and solve simultaneously – look for them reaching a value for *p*.

A1: Correct value for *p* or *x*

M1: Must have been attempting $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0$ to find *p*. Substitutes back into the curve to find *k*. May be scored if their numerator was in terms of *x* only.

A1: Correct value for *k*

Alt (b)

M1: Substitutes (*p*, 2) into the equation of the curve and attempts to complete the square or considers the discriminant (or other valid method to be able to deduce the values using a single root)

A1: Correct value for *p* or *k*

M1: Substitutes back into the curve to find the remaining unknown or other complete method to find the other value.

A1: Correct value for the other of *p* and *k* (so both should be correct).

Question	Scheme	Marks
4(a)	$(l=)\sqrt{25+r^2}$	**B1**
		(1)
(b)	$\left(S=\pi r^2+\right)\pi r\sqrt{25+r^2} \Rightarrow \left(\frac{\mathrm{d}S}{\mathrm{d}r}=2\pi r+\right)\underline{\pi\sqrt{25+r^2}+\pi r.\frac{1}{2}\left(25+r^2\right)^{-\frac{1}{2}}.2r}$ Or $\left(S=\pi r^2+\right)\pi\sqrt{25r^2+r^4} \rightarrow \left(\frac{\mathrm{d}S}{\mathrm{d}r}=2\pi r+\right)\underline{\pi.\frac{1}{2}\left(25r^2+r^4\right)^{-\frac{1}{2}}\times\left(50r+4r^3\right)}$	**M1** **A1**
	$\left(S=\pi\left(l^2-25\right)\right)+\pi l\sqrt{l^2-25} \Rightarrow \left(\frac{\mathrm{d}S}{\mathrm{d}l}=2\pi l+\right)\underline{\pi\sqrt{l^2-25}+\pi l.\frac{1}{2}\left(l^2-25\right)^{-\frac{1}{2}}.2l}$	**(M1** **A1)**
	$\frac{\mathrm{d}S}{\mathrm{d}t}=\frac{\mathrm{d}S}{\mathrm{d}r}\times\frac{\mathrm{d}r}{\mathrm{d}t}=..\times 3$	**M1**
	$=81.5\left(\mathrm{cm}^2/\mathrm{min}\right)$	**A1**
		(4)
		(5 marks)

Notes:

(a)

B1: Correct expression. Accept $(l=)\sqrt{5^2+r^2}$ and isw.

(b)

M1: Differentiates their surface area with use of the product and chain rules on a term of the form $Kr\sqrt{\pm L\pm r^2}$ - look the form (oe) $P\sqrt{"25"+r^2}+Qr^2\left("25"+r^2\right)^{-\frac{1}{2}}$. The base area may be missing for this mark. Alternatively they may work in terms of *l* and the scheme marks equivalently – see box in main scheme for the derivative in this case.

Alt: Taking *r* inside the square root look for $K\sqrt{25r^2+r^4} \rightarrow K.\frac{1}{2}\left(25r^2+r^4\right)^{-\frac{1}{2}}\times\left(\alpha r+\beta r^3\right)$

A1: Correct differentiation for the curved surface area. Need not be simplified.

Alt: With *r* inside the square root $\pi\sqrt{25r^2+r^4} \rightarrow \pi.\frac{1}{2}\left(25r^2+r^4\right)^{-\frac{1}{2}}\times\left(50r+4r^3\right)$

M1: Applies the chain rule correctly with $\frac{\mathrm{d}r}{\mathrm{d}t}=3$ and their expression for $\frac{\mathrm{d}S}{\mathrm{d}r}$ (which may be in terms of *r* and *l*) or their value for $\frac{\mathrm{d}S}{\mathrm{d}r}$ at *r* = 1.5 if they find this first. This mark is for the chain rule being applied, so their previous expressions need not have been correct. If they are working in terms of *l* they will need to use $\frac{\mathrm{d}S}{\mathrm{d}t}=\frac{\mathrm{d}S}{\mathrm{d}l}\times\frac{\mathrm{d}l}{\mathrm{d}r}\times\frac{\mathrm{d}r}{\mathrm{d}t}$.