| Question | Scheme | Marks |
|---|---|---|
| **2(a)** | $\dfrac{3x+4}{(x-2)(2x+1)^2}=\dfrac{A}{x-2}+\dfrac{B}{2x+1}+\dfrac{C}{(2x+1)^2}$ | |
| | $3x+4=A(2x+1)^2+B(x-2)(2x+1)+C(x-2)$<br>E.g. $x=2\rightarrow 10=A\times 25\Rightarrow A=...$ or $x=-\dfrac{1}{2}\rightarrow \dfrac{5}{2}=C\times-\dfrac{5}{2}\Rightarrow C=...$<br>or $x^2:0=4A+2B, x:3=4A-3B+C, x^0:4=A-2B-2C\rightarrow A=...$ | **M1** |
| | One of $A=\dfrac{2}{5}, B=-\dfrac{4}{5}, C=-1$ | **A1** |
| | $\Rightarrow A=..., B=...$ and $C=...$ | **M1** |
| | $\dfrac{3x+4}{(x-2)(2x+1)^2}=\dfrac{2}{5(x-2)}-\dfrac{4}{5(2x+1)}-\dfrac{1}{(2x+1)^2}$ or $A=\dfrac{2}{5}, B=-\dfrac{4}{5}, C=-1$ | **A1** |
| | | **(4)** |
| **(b)** | $\int\dfrac{p}{x-2}\,\mathrm{d}x=\alpha\ln(x-2)$ or $\int\dfrac{q}{2x+1}\,\mathrm{d}x=\beta\ln(2x+1)$<br>Alt: $\int\dfrac{8x+9}{(2x+1)^2}\,\mathrm{d}x=\int\dfrac{8x+4}{(2x+1)^2}+\dfrac{5}{(2x+1)^2}\,\mathrm{d}x=\gamma\ln(2x+1)^2+...$ | **M1** |
| | $\int\dfrac{k}{(2x+1)^2}\,\mathrm{d}x=\dfrac{K}{2x+1}$ | **M1** |
| | $\int_7^{12}\dfrac{3x+4}{(x-2)(2x+1)^2}\,\mathrm{d}x=\left[\text{"}\dfrac{2}{5}\text{"}\ln(x-2)-\text{"}\dfrac{4}{5}\text{"}\times\dfrac{1}{2}\ln(2x+1)+\dfrac{\text{"1"}}{4x+2}\right]_7^{12}$ | **A1ft** |
| | $=\left(\dfrac{2}{5}\ln(10)-\dfrac{2}{5}\ln(25)+\dfrac{1}{50}\right)-\left(\dfrac{2}{5}\ln(5)-\dfrac{2}{5}\ln(15)+\dfrac{1}{30}\right)=...$ | **DM1** |
| | $=\dfrac{2}{5}\ln\dfrac{10}{25}+\dfrac{1}{50}-\dfrac{2}{5}\ln\dfrac{5}{15}-\dfrac{1}{30}=\dfrac{2}{5}\ln\dfrac{10\times15}{25\times5}+...$ | **M1** |
| | $\dfrac{2}{5}\ln\dfrac{6}{5}-\dfrac{1}{75}$ (oe) | **A1** |
| | | **(6)** |
| | | **(10 marks)** |

**Notes:**

**(a)**

**M1:** Attempts a correct method leading to one of the constants. E.g. Multiplies through to $3x+4=A(2x+1)^2+B(x-2)(2x+1)+C(x-2)$ (allowing for minor slips) and substitutes $x=-\frac{1}{2}$ or $x = 2$ or other method such as comparing coefficients. Implied by a correct value for *A* by cover-up rule if no incorrect working is seen. If e.g. multiplies through by a wrong term such as $(2x+1)^3$ then allow M1 here for the attempt.

**A1:** Any one of the three constants correct (probably *A* or *C*).

**M1:** For a full and correct method to obtain all three of the constants. If e.g. multiplies through by a wrong term such as $(2x+1)^3$ then M0 here. Implied by correct values if no method shown following a correct identity (otherwise M0 for just incorrect values stated).

**A1:** For a correct full partial expression form or correct values stated.

Note: if by comparing coefficients allow both M's for achieving any 3 values following three equations from the attempt at comparing with at least one correct.

**(b)**

**M1:** For integrating to obtain at least one ln term. Note $\alpha\ln(5x-10)$ and $\beta\ln(10x+5)$ are equally fine, as are other constant multiples of the inner brackets.

**M1:** For $\frac{k}{(2x+1)^2}\to\frac{K}{(2x+1)}$ (any *K*) oe.

**A1ft:** Fully correct integration following through on their non-zero constants.

**DM1:** Depends on at least one the first two M's being scored. Substitutes the limits and subtracts either way round.

**M1:** Achieves a single log term with evidence of a correct use of at least one log law (e.g. power or addition law). (Note log terms may initially have been combined earlier correctly, in which case look for reaching a single log term.)

**A1:** Correct answer accept equivalents in the correct form (including if the modulus signs are left in).

Use of constants (if no values for *A*, *B* and *C* are found) can score the M's but not the A's.

| Question | Scheme | Marks |
|---|---|---|
| **3(a)** | $y^2x+3y=4x^2+k$ | |
| | $y^2x \to ...\times y\dfrac{\mathrm{d}y}{\mathrm{d}x}(+...)$ or $3y \to 3\dfrac{\mathrm{d}y}{\mathrm{d}x}$ | **M1** |
| | $y^2x \to 2xy\dfrac{\mathrm{d}y}{\mathrm{d}x}+y^2$ | **A1** |
| | $...+3y=4x^2+k \to ...+3\dfrac{\mathrm{d}y}{\mathrm{d}x}=8x$ | **A1** |
| | $2xy\dfrac{\mathrm{d}y}{\mathrm{d}x}+y^2+3\dfrac{\mathrm{d}y}{\mathrm{d}x}=8x \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x}=...$ | **dM1** |
| | $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{8x-y^2}{2xy+3}$ oe | **A1** |
| | | **(5)** |
| **(b)** | $\dfrac{\mathrm{d}y}{\mathrm{d}x}=0 \Rightarrow 8x-4=0 \Rightarrow x=...$ | **M1** |
| | $p=\dfrac{1}{2}$ | **A1** |
| | $k=2^2\times\dfrac{1}{2}+3\times2-4\times\dfrac{1}{4}=...$ | **M1** |
| | $k=7$ | **A1** |
| | | **(4)** |
| **Alt (b)** | $(p,2)$ on $C \Rightarrow 4p+6=4p^2+k \Rightarrow 4p^2-4p+k-6=0$<br>$\Rightarrow 4\left(p-\dfrac{1}{2}\right)^2-1+k-6=0$ or $b^2-4ac=16-16(k-6)$ | **M1** |
| | (Min, single solution) $\Rightarrow p=\dfrac{1}{2}$ or $16-16(k-6)=0 \Rightarrow k=7$ | **A1** |
| | $\Rightarrow k=7-4\left(\dfrac{1}{2}-\dfrac{1}{2}\right)^2$ or $4p^2-4p+7-6=0 \Rightarrow p=...$ | **M1** |
| | The other of $p=\dfrac{1}{2}$ and $k=7$ | **A1** |
| | | **(4)** |
| | | **(9 marks)** |

**Notes:**

**(a)**

**M1:** Some evidence of an attempt at implicit differentiation. Look for one of the terms being differentiated with either the $3\dfrac{\mathrm{d}y}{\mathrm{d}x}$ or a term with $kxy\dfrac{\mathrm{d}y}{\mathrm{d}x}$ or $y^2 \to ky\dfrac{\mathrm{d}y}{\mathrm{d}x}$ as part of a product rule seen appropriately.

**A1:** $y^2x$ differentiated correctly