Question Number	Scheme	Marks
7(a)	Substitutes $x = 3$ and $t = 0$ into $x = \dfrac{k(9t+5)}{4t+3}$	M1
	$\Rightarrow 3 = \dfrac{5k}{3} \Rightarrow k = 1.8$ *	A1*
		(2)
(b)	4050	B1
		(1)
(c)	$\dfrac{3}{x(9-2x)} \equiv \dfrac{A}{x} + \dfrac{B}{9-2x}$	M1
	Either $A = \dfrac{1}{3}$ or $B = \dfrac{2}{3}$	A1
	$\dfrac{3}{9x-2x^2} \equiv \dfrac{1}{3x} + \dfrac{2}{3(9-2x)}$	A1
		(3)
(d)	$3\dfrac{\mathrm{d}x}{\mathrm{d}t} = x(9-2x) \Rightarrow \displaystyle\int \frac{3}{x(9-2x)}\,\mathrm{d}x = \int \mathrm{d}t$ **Or** $3\dfrac{\mathrm{d}x}{\mathrm{d}t} = x(9-2x) \Rightarrow \displaystyle\int \frac{1}{x(9-2x)}\,\mathrm{d}x = \int \frac{1}{3}\,\mathrm{d}t$ $\Rightarrow \dfrac{1}{3}\ln x - \dfrac{1}{3}\ln(9-2x) = t(+c)$ **or** $\dfrac{1}{9}\ln x - \dfrac{1}{9}\ln(9-2x) = \dfrac{1}{3}t(+c)$	M1 A1ft
	Substitutes $t = 0, x = 3 \Rightarrow c = 0$	M1
	$\dfrac{1}{3}\ln x - \dfrac{1}{3}\ln(9-2x) = t \Rightarrow \left(\dfrac{x}{9-2x}\right) = \mathrm{e}^{3t}$	**dd**M1
	$\left(\dfrac{9-2x}{x}\right) = \mathrm{e}^{-3t} \Rightarrow \dfrac{9}{x} - 2 = \mathrm{e}^{-3t} \Rightarrow x = \dfrac{9}{2+\mathrm{e}^{-3t}}$*	A1*
		(5)
(e)	4500	B1
		(1)
		(12 marks)

(a)

M1: Substitutes $x = 3$ and $t = 0$ into the given equation

A1*: Shows that $k = 1.8$ (oe e.g. $\dfrac{9}{5}$) with no errors and with at least one correct line of the form $ak = b$

e.g. $3 = \dfrac{k(9\times 0+5)}{4\times 0+3} \Rightarrow k = 1.8$, $3 = \dfrac{k(0+5)}{0+3} \Rightarrow k = 1.8$, $3 = \dfrac{5k}{3} \Rightarrow k = 1.8$ **all score M1A1**

Alternative by verification:

M1: Substitutes $k = 1.8$ and $t = 0$ into the given equation

A1*: Shows that $x = 3$ with at least one correct intermediate line and a concluding statement that this is 3000

Special cases:

1. $x = \dfrac{k(9\times 0+5)}{4\times 0+3} \Rightarrow k = 1.8$ scores M1A0

2. Substitutes $x = 3000$ and $t = 0$ into the given equation to obtain $k = 1800$ (hence $k = 1.8$) scores M1A0

(b)

B1: 4050 cao. Allow 4.05 thousand.

(c)

M1: Sets $\dfrac{3}{x(9-2x)} \equiv \dfrac{A}{x} + \dfrac{B}{9-2x}$ or equivalent e.g. $3 \equiv A(9-2x) + Bx$

A1: One correct value for 'A' or 'B' or one correct fraction.

A1: Correct **fractions** e.g. $\dfrac{3}{x(9-2x)} \equiv \dfrac{1}{3x} + \dfrac{2}{3(9-2x)}$ oe e.g. $\dfrac{1\!/\!3}{x} + \dfrac{2\!/\!3}{(9-2x)}$, $\dfrac{1}{3x} + \dfrac{2}{27-6x}$

This mark is for the correct partial fractions not for the values of the constants.

Award once the correct fractions are seen and allow if seen in (d) if not seen here.

(d)

M1: Attempts to separate the variables and integrate to obtain a kt term and one of $\alpha \ln \beta x$ or $\gamma \ln \delta(9-2x)$.

Condone missing brackets e.g. $\ln 9-2x$

The ln terms must come from a partial fraction of the form $\dfrac{A}{x}$ or $\dfrac{B}{9-2x}$.

A1ft: $\dfrac{1}{3}\ln x - \dfrac{1}{3}\ln(9-2x) = t\,(+c)$ but ft on their A and B so award for $A\ln x - \dfrac{B}{2}\ln(9-2x) = t\,(+c)$ oe

There is no requirement for + c

Brackets must be present unless they are implied by later work.

Note that there are various correct alternatives here e.g. $\dfrac{1}{3}\ln 3x - \dfrac{1}{3}\ln(27-6x) = t\,(+c)$

M1: Sets $t = 0$ and $x = 3$ in an attempt to find "c" having made some attempt to integrate at least one of their partial fractions to obtain a ln term.

If this step is not attempted only the first two marks are available in this part.

They can "state" e.g. $c = 0$ or e.g. $K = 1$ provided it follows correct work and there was a "+ c".

ddM1: **Dependent upon both previous Method marks.**

It is for using correct work to remove the ln's having found a constant of integration.

A1*: Correct work to reach the given answer showing all necessary steps.

The scheme shows one such way with acceptable minimal working.

Note that some candidates may rearrange first before finding the constant of integration e.g.

$$3\frac{\mathrm{d}x}{\mathrm{d}t} = x\left(9-2x\right) \Rightarrow \int \frac{3}{x\left(9-2x\right)}\mathrm{d}x = \int \mathrm{d}t$$

$$\Rightarrow \frac{1}{3}\ln x - \frac{1}{3}\ln\left(9-2x\right) = t + c$$

$$\Rightarrow \frac{1}{3}\ln\frac{x}{9-2x} = t + c \Rightarrow \ln\frac{x}{9-2x} = 3t + d \Rightarrow \frac{x}{9-2x} = K\mathrm{e}^{3t}$$

$$t = 0 \text{ and } x = 3 \Rightarrow K = 1$$

$$\frac{x}{9-2x} = \mathrm{e}^{3t} \Rightarrow 9\mathrm{e}^{3t} - 2x\mathrm{e}^{3t} = x \Rightarrow x\left(2\mathrm{e}^{3t}+1\right) = 9\mathrm{e}^{3t}$$

$$\Rightarrow x = \frac{9\mathrm{e}^{3t}}{\left(2\mathrm{e}^{3t}+1\right)} = \frac{9}{2+\mathrm{e}^{-3t}}*$$

In these cases, the first 2 marks are as already defined and then award M1 at the point $t = 0$ and $x = 3$ are used in an attempt to find "c" and then **dd**M1 for using correct work to remove the ln's and A1* for correct work to reach the given answer showing all necessary steps.

As in the main scheme, if there is no "+ *c*" then only the first 2 marks are available in (d).

Note that in (d) it is possible to start again e.g.

$$3\frac{\mathrm{d}x}{\mathrm{d}t} = x\left(9-2x\right) \Rightarrow \int \frac{1}{x\left(9-2x\right)}\mathrm{d}x = \int \frac{1}{3}\mathrm{d}t$$

$$\frac{1}{x(9-2x)} \equiv \frac{A}{x} + \frac{B}{9-2x} \equiv \frac{1}{9x} + \frac{2}{9(9-2x)}$$

etc.

(e)

B1: 4500 cao or 4.5 thousand

Question Number	Scheme	Marks
8(a)	3π	B1
		(1)
(b)	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{-2\sin t}{6-6\cos 2t}$	M1 A1
	$= \dfrac{-2\sin t}{6-6\left(1-2\sin^2 t\right)} = \dfrac{-2\sin t}{12\sin^2 t} = -\dfrac{1}{6}\operatorname{cosec} t$	M1 A1*
		(4)
(c)	$P = \left(\dfrac{3\pi}{2}-3, \sqrt{2}\right)$	B1
	E.g. $y-\sqrt{2} = -\dfrac{\sqrt{2}}{6}\left(0-\left(\dfrac{3\pi}{2}-3\right)\right) \Rightarrow y = \ldots$ or $\sqrt{2} = -\dfrac{\sqrt{2}}{6}\left(\dfrac{3\pi}{2}-3\right)+c \Rightarrow c = \ldots$	M1
	$\dfrac{\pi\sqrt{2}}{4}+\dfrac{\sqrt{2}}{2}$ o.e	A1
		(3)
(d)(i)	Attempts $y^2\dfrac{\mathrm{d}x}{\mathrm{d}t} = 4\cos^2 t\left(6-6\cos 2t\right)$	M1 A1
	$= \left(2\cos 2t+2\right)\left(6-6\cos 2t\right) = 6\left(2-2\cos^2 2t\right) = 6\left(2-\left(1+\cos 4t\right)\right)$	**d**M1
	Volume $= \displaystyle\int_0^{\frac{\pi}{2}} \pi y^2 \frac{\mathrm{d}x}{\mathrm{d}t}\,\mathrm{d}t = \int_0^{\frac{\pi}{2}} 6\pi\left(1-\cos 4t\right)\mathrm{d}t$	A1
(ii)	$\left[6\pi t - \dfrac{6\pi\sin 4t}{4}\right]_0^{\frac{\pi}{2}}, = 3\pi^2$	M1, A1
		(6)
		(14 marks)

(a)

B1: For 3π. Condone $\left(3\pi, 0\right)$ and allow $x = 3\pi$. Remember to check the diagram as the 3π may appear there.

Do not allow decimals here.

(b)

M1: Attempts to differentiate $x(t)$ and $y(t)$ and calculates $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ by using $\dfrac{\mathrm{d}y}{\mathrm{d}t} \div \dfrac{\mathrm{d}x}{\mathrm{d}t}$.

Condone poor differentiation but one of the terms must be of the correct form so it requires $x \to \alpha + \beta\cos 2t$ or $y \to A\sin t$

A1: $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{-2\sin t}{6-6\cos 2t}$ or equivalent e.g. $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{-2\sin t}{6-6\left(\cos^2 t-\sin^2 t\right)}$