7. The number of goats on an island is being monitored.

When monitoring began there were 3000 goats on the island.

In a simple model, the number of goats, x, in thousands, is modelled by the equation

$$x = \frac{k(9t + 5)}{4t + 3}$$

where k is a constant and t is the number of years after monitoring began.

(a) Show that $k = 1.8$

(2)

(b) Hence calculate the long-term population of goats predicted by this model.

(1)

In a **second** model, the number of goats, x, in thousands, is modelled by the differential equation

$$3\frac{dx}{dt} = x(9 - 2x)$$

(c) Write $\frac{3}{x(9 - 2x)}$ in partial fraction form.

(3)

(d) Solve the differential equation with the initial condition to show that

$$x = \frac{9}{2 + e^{-3t}}$$

(5)

(e) Find the long-term population of goats predicted by this **second** model.

(1)

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Question 7 continued

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Question 7 continued

(Total for Question 7 is 12 marks)

8.

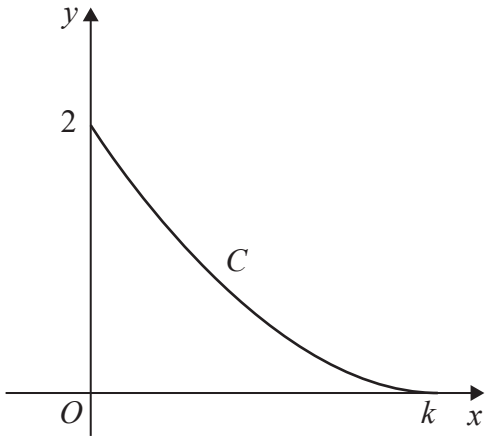


Figure 3

Figure 3 shows a sketch of the curve C with parametric equations

$$x = 6t - 3\sin 2t \qquad y = 2\cos t \qquad 0 \leqslant t \leqslant \frac{\pi}{2}$$

The curve meets the y-axis at 2 and the x-axis at k, where k is a constant.

(a) State the value of k.

(1)

(b) Use parametric differentiation to show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \lambda \operatorname{cosec} t$$

where λ is a constant to be found.

(4)

The point P with parameter $t = \frac{\pi}{4}$ lies on C.

The tangent to C at the point P cuts the y-axis at the point N.

(c) Find the exact y coordinate of N, giving your answer in simplest form.

(3)

The region bounded by the curve, the x-axis and the y-axis is rotated through 2π radians about the x-axis to form a solid of revolution.

(d) (i) Show that the volume of this solid is given by

$$\int_0^{\alpha} \beta\left(1 - \cos 4t\right) \mathrm{d}t$$

where α and β are constants to be found.

(ii) Hence, using algebraic integration, find the exact volume of this solid.

(6)

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