6. The line l_1 has equation $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ -7 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$ where λ is a scalar parameter.

The line l_2 has equation $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ -7 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ -1 \\ 8 \end{pmatrix}$ where μ is a scalar parameter.

Given that l_1 and l_2 meet at the point P

(a) state the coordinates of P

(1)

Given that the angle between lines l_1 and l_2 is θ

(b) find the value of $\cos\theta$, giving the answer as a fully simplified fraction.

(3)

The point Q lies on l_1 where $\lambda = 6$

Given that point R lies on l_2 such that triangle QPR is an isosceles triangle with $PQ = PR$

(c) find the exact area of triangle QPR

(3)

(d) find the coordinates of the possible positions of point R

(3)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 6 continued

Question 6 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 6 continued

(Total for Question 6 is 10 marks)

7. The number of goats on an island is being monitored.

When monitoring began there were 3000 goats on the island.

In a simple model, the number of goats, x, in thousands, is modelled by the equation

$$x = \frac{k(9t + 5)}{4t + 3}$$

where k is a constant and t is the number of years after monitoring began.

(a) Show that $k = 1.8$

(2)

(b) Hence calculate the long-term population of goats predicted by this model.

(1)

In a **second** model, the number of goats, x, in thousands, is modelled by the differential equation

$$3\frac{dx}{dt} = x(9 - 2x)$$

(c) Write $\frac{3}{x(9 - 2x)}$ in partial fraction form.

(3)

(d) Solve the differential equation with the initial condition to show that

$$x = \frac{9}{2 + e^{-3t}}$$

(5)

(e) Find the long-term population of goats predicted by this **second** model.

(1)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA