Question Number	Scheme	Marks
4(a)	Assume that there exists a positive number k such that $k+\frac{9}{k}<6$	B1
	$k+\frac{9}{k}<6 \Rightarrow k^2+9<6k \Rightarrow k^2-6k+9<0$ **or** $k+\frac{9}{k}<6 \Rightarrow k+\frac{9}{k}-6<0 \Rightarrow \left(\sqrt{k}...\right)\left(\sqrt{k}...\right)<0$ **or** $k+\frac{9}{k}<6 \Rightarrow \left(k+\frac{9}{k}\right)^2<36 \Rightarrow k^2+18+\frac{81}{k^2}-36<0$	M1
	$\Rightarrow \left(k-3\right)^2<0$ **or** $\Rightarrow \left(\sqrt{k}-\frac{3}{\sqrt{k}}\right)^2<0$ **or** $\Rightarrow \left(k-\frac{9}{k}\right)^2<0$	A1
	But numbers squared are $\geqslant 0$, hence $k+\frac{9}{k}\geqslant 6$	A1*
		(4)
(b)	E.g. When $k=-3$, $-3+\frac{9}{-3}\left(=-6\right)$ which is not $\geqslant 6$	B1
		(1)
(b) Alt	$k<0 \Rightarrow \frac{9}{k}<0 \Rightarrow k+\frac{9}{k}<0$ which is not $\geqslant 6$	B1
		(5 marks)

(a)

B1: For setting up the correct contradiction. It must include a word/words such as "assume" or "let" and must be a strict inequality. As a minimum accept just "assume/let $k+\frac{9}{k}<6$".

Condone e.g. Assume that for all positive numbers k, $k+\frac{9}{k}<6$

M1: Starting from $k+\frac{9}{k}<6$ or $k+\frac{9}{k}\leqslant 6$ or $k+\frac{9}{k}=6$, either

- multiplies by k and attempts to reach $k^2...<0$ or $k^2...\leqslant 0$ or $k^2...=0$
- collects terms to one side to reach $k+\frac{9}{k}\pm 6 \leqslant / < / = 0$ and attempts to factorise to $\left(\sqrt{k}...\right)\left(\sqrt{k}...\right)\leqslant / < / = 0$
- squares both sides and collects terms to reach $k^2...<0$ or $k^2...\leqslant 0$ or $k^2...=0$

A1: Reaches $\left(k-3\right)^2...0$ or e.g. $\left(\sqrt{k}-\frac{3}{\sqrt{k}}\right)^2...0$ or e.g. $\left(k-\frac{9}{k}\right)^2...0$ where … is < or $\leqslant$ or =

A1*: For a fully correct proof.
Requires

- correct calculations/algebra
- a reason: "numbers squared are $\geqslant 0$ "this is impossible", "numbers squared are not negative"
- a minimal conclusion. E.g. hence $k+\frac{9}{k}\geqslant 6$, hence proven, QED, etc. but not just "contradiction"

<u>Alternatives for A1A1*:</u>

1. Using discriminant:

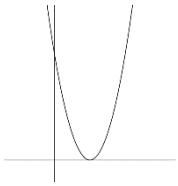
A1: $b^2-4ac=6^2-4\times 9=0$ so one root and 'positive' quadratic

A1*: For a fully correct proof.
Requires

- correct calculations/algebra
- a reason e.g. so $k^2-6k+9\geqslant 0$ or curve is on or above the *x*/*k* axis
- a minimal conclusion. E.g. hence $k+\frac{9}{k}\geqslant 6$, hence proven, QED, etc. but not just "contradiction"

2. Via sketch:

A1: Sketches $y=k^2-6k+9$:



A1*: For a fully correct proof.
Requires

- correct calculations/algebra
- a reason e.g. graph always on or above (or never below) the *x*-axis so $k^2-6k+9\geqslant 0$
- a minimal conclusion. E.g. hence $k+\frac{9}{k}\geqslant 6$, hence proven, QED, etc. but not just "contradiction"

3. Via differentiation:

A1: $\frac{\mathrm{d}\left(k^2-6k+9\right)}{\mathrm{d}k}=2k-6=0\Rightarrow k=3\Rightarrow k^2-6k+9=0$ so minimum at (3, 0)

A1*: For a fully correct proof.
Requires

- correct calculations/algebra
- a reason e.g. so $k^2-6k+9\geqslant 0$ or curve is on or above the *x*/*k* axis
- a minimal conclusion. E.g. hence $k+\frac{9}{k}\geqslant 6$, hence proven, QED, etc. but not just "contradiction"

If candidates use another variable for *k*, e.g. *x* or *m* then withhold the final mark unless they revert to *k* in their conclusion.

This is also acceptable (working backwards):

Assume $k+\frac{9}{k}<6$ B1

$\left(k-3\right)^2 \geqslant 0 \Rightarrow k^2-6k+9 \geqslant 0 \Rightarrow k-6+\frac{9}{k} \geqslant 0$ M1

Condone > for $\geqslant$

From $\left(k-3\right)^2 \geqslant 0$ expands and divides by k

$\Rightarrow k+\frac{9}{k} \geqslant 6$ A1

Condone > for $\geqslant$

This contradicts the assumption and so $k+\frac{9}{k} \geqslant 6$ A1*

Note: Attempts that use the contradiction: "Assume there are negative numbers for which $k+\frac{9}{k} \geqslant 6$" generally score no marks but use review if necessary.

(b)

B1: Chooses a negative number, shows the result of substituting into $k+\frac{9}{k}$ and gives a minimal conclusion e.g. It is not necessary to evaluate e.g. $-3+\frac{9}{-3}$ is sufficient.

- which is not $\geqslant 6$
- is < 6

(b)**Alt:**

B1: States that if $k < 0$ then $k+\frac{9}{k}<0$ and gives a minimal conclusion e.g.

- which is not $\geqslant 6$
- is < 6

There must be no incorrect work and no contradictory/incorrect statements.
Do not allow arguments that refer to *k* = 0 being not valid.

Question Number	Scheme	Marks
5(a)	$y^3 - x^2 + 4x^2y = k$	
	$y^3 \to 3y^2\dfrac{dy}{dx}$	M1
	$4x^2y \to 8xy + 4x^2\dfrac{dy}{dx}$	M1
	$y^3 - x^2 + 4x^2y = k \Rightarrow 3y^2\dfrac{dy}{dx} - 2x + 8xy + 4x^2\dfrac{dy}{dx} = 0$	A1
	$\Rightarrow \left(3y^2 + 4x^2\right)\dfrac{dy}{dx} = 2x - 8xy$	M1
	$\Rightarrow \dfrac{dy}{dx} = \dfrac{2x - 8xy}{3y^2 + 4x^2}$	A1
		(5)
(b)	$\dfrac{dy}{dx} = -1$ at P	M1
	Uses $\dfrac{dy}{dx} = \pm 1$ and $y = x$ to set up and solve equation in x, y or 'p' $\Rightarrow -1 = \dfrac{2p - 8p^2}{3p^2 + 4p^2} \Rightarrow p = 2$	M1 A1
	e.g. $k = "2"^3 - "2"^2 + 4 \times "2"^3$	**dd**M1
	$k = 36$	A1
		(5)
		(10 marks)

(a) Allow equivalent notation for the $\dfrac{dy}{dx}$ e.g. y'

M1: Differentiates $y^3 \to Py^2\dfrac{dy}{dx}$

M1: Uses the product rule to differentiate $4x^2y$ and obtains $Qxy + Rx^2\dfrac{dy}{dx}$

A1: Correct differentiation including the " = 0" which may be implied by subsequent work.

Note that some candidates have a spurious $\dfrac{dy}{dx} = ...$ at the start (as their intention to differentiate) and this can be ignored for the first 3 marks so condone $\dfrac{dy}{dx} = 3y^2\dfrac{dy}{dx} - 2x + 8xy + 4x^2\dfrac{dy}{dx} = 0$

Allow versions such as $3y^2dy - 2xdx + 8xydx + 4x^2dy = 0$

M1: Dependent upon having achieved two <u>different</u> terms in $\dfrac{dy}{dx}$, one from each of the terms y^3 and $4x^2y$.

Look for $(... \pm ...)\dfrac{dy}{dx} = ... \Rightarrow \dfrac{dy}{dx} = ...$ which may be implied by their working.

For those candidates who had a spurious $\dfrac{dy}{dx} = ...$ at the start, they may incorporate this in their rearrangement in which case they will have 3 terms in $\dfrac{dy}{dx}$ and so score M0.

If they ignore it, then this mark is available for the condition as described above.

A1: $\dfrac{dy}{dx} = \dfrac{2x - 8xy}{3y^2 + 4x^2}$ or exact equivalent e.g. $\dfrac{dy}{dx} = \dfrac{8xy - 2x}{-3y^2 - 4x^2}$