(ii)	**1.** $u = 2x-1 \Rightarrow \dfrac{\mathrm{d}u}{\mathrm{d}x} = 2$ or e.g. $\mathrm{d}u = 2\mathrm{d}x$ **OR** **2.** $u = \left(2x-1\right)^2 \Rightarrow \dfrac{\mathrm{d}u}{\mathrm{d}x} = 4\left(2x-1\right)$ or e.g. $\mathrm{d}u = 4\left(2x-1\right)\mathrm{d}x$ **OR** **3.** $u = 2x \Rightarrow \dfrac{\mathrm{d}u}{\mathrm{d}x} = 2$ or e.g. $\mathrm{d}u = 2\mathrm{d}x$	B1
	1. $u = 2x-1 \Rightarrow \displaystyle\int \frac{4x}{\left(2x-1\right)^2}\,\mathrm{d}x = \int \frac{2u+2}{u^2} \times \frac{1}{2}\,\mathrm{d}u$ **OR** **2.** $u = \left(2x-1\right)^2 \Rightarrow \displaystyle\int \frac{4x}{\left(2x-1\right)^2}\,\mathrm{d}x = \int \frac{2\sqrt{u}+2}{u} \times \frac{1}{4\sqrt{u}}\,\mathrm{d}u$ **OR** **3.** $u = 2x \Rightarrow \displaystyle\int \frac{4x}{\left(2x-1\right)^2}\,\mathrm{d}x = \int \frac{2u}{\left(u-1\right)^2} \times \frac{1}{2}\,\mathrm{d}u$	M1 A1
	1. $\displaystyle\int \left(\frac{1}{u} + \frac{1}{u^2}\right)\mathrm{d}u = \ln u - \frac{1}{u}$ **OR** **2.** $\displaystyle\frac{1}{2}\int \left(\frac{1}{u} + \frac{1}{u^{\frac{3}{2}}}\right)\mathrm{d}u = \frac{1}{2}\ln u - \frac{1}{\sqrt{u}}$ **OR 3.** $\displaystyle\int \frac{u}{\left(u-1\right)^2}\,\mathrm{d}u = -u\left(u-1\right)^{-1} + \int \frac{1}{u-1}\mathrm{d}u = -u\left(u-1\right)^{-1} + \ln\left(u-1\right)$	**d**M1 A1
	1. Uses limits $u = 5$ to $u = 20$: $\left(\ln 20 - \dfrac{1}{20}\right) - \left(\ln 5 - \dfrac{1}{5}\right)$ **OR** **2.** Uses limits $u = 25$ to $u = 400$: $\left(\dfrac{1}{2}\ln 400 - \dfrac{1}{20}\right) - \left(\dfrac{1}{2}\ln 25 - \dfrac{1}{5}\right)$ **OR** **3.** Uses limits $u = 6$ to $u = 21$: $\left(\ln 20 - \dfrac{21}{20}\right) - \left(\ln 5 - \dfrac{6}{5}\right)$	M1
	$= \dfrac{3}{20} + \ln 4$	A1
		(7)
		(12 marks)

Note that 'u' = $2x - 1$ is by far the most common approach.

(ii)

B1: Uses an appropriate substitution and differentiates **correctly**. See scheme for 3 examples.

Note that for case **2.** $u=\left(2x-1\right)^2 \Rightarrow x=\dfrac{\sqrt{u}+1}{2} \Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}u}=\dfrac{1}{4}u^{-\frac{1}{2}}$ is also correct and scores B1

M1: Valid attempt to change the integral to one in "u" to obtain an integrand of the correct form:

This requires e.g.

1. $k\displaystyle\int \frac{au+b}{u^2}\mathrm{d}u,\ a,b\neq 0$ **OR** **2.** $k\displaystyle\int \frac{a\sqrt{u}+b}{u}\frac{1}{\sqrt{u}}\mathrm{d}u,\quad a,b\neq 0$ **OR** **3.** $k\displaystyle\int \frac{u}{\left(u-1\right)^2}\mathrm{d}u$

Condone the omission of du as long as the form of the integrand is correct.

Note that the u's may appear "combined" e.g. as $u^{\frac{3}{2}}$ in the denominator of case **2.**

A1: A correct integrand in any form in terms of u only.

Condone the omission of du as long as the integrand is correct.

dM1: Correct form of the integration for their substitution: This requires e.g.

1. $k\displaystyle\int \frac{au+b}{u^2}\mathrm{d}u \rightarrow p\ln u+qu^{-1}$ **OR** **2.** $k\displaystyle\int \frac{a\sqrt{u}+b}{u}\frac{1}{\sqrt{u}}\mathrm{d}u \rightarrow p\ln u+\frac{q}{\sqrt{u}}$

OR **3.** $k\displaystyle\int \frac{u}{\left(u-1\right)^2}\mathrm{d}u \rightarrow \alpha u\left(u-1\right)^{-1}+\beta\ln\left(u-1\right)$

Depends on the previous method mark.

A1: Fully correct integration. Ignore any limits for this mark.

Note that there may be other acceptable attempts to integrate e.g. in case **1.**:

$$\int \frac{u+1}{u^2}\mathrm{d}u=\int \left(u+1\right)u^{-2}\mathrm{d}u=-\left(u+1\right)\frac{1}{u}+\int \frac{1}{u}\mathrm{d}u=-\left(u+1\right)\frac{1}{u}+\ln u$$

In such cases, award M1 for a fully correct method and A1 if correct.

M1: Attempts to use the correct limits for their substitution.

Need to see evidence that both correct changed limits have been substituted and the resulting expressions subtracted either way round.

There must have been some attempt to integrate however poor having used substitution.

The substitution must involve a transformation so e.g. $u = x$ is not acceptable.

Alternatively, reverses the substitution and uses the x limits 3 and $\dfrac{21}{2}$.

A1: $\ln 4+\dfrac{3}{20}$ or e.g. $\ln 4+0.15$ and apply isw if necessary.

Special Case: Candidates who answer part (b) without any form of substitution can score a special case of B0M0A0 (no substitution) then M1A1dM1A1

e.g. Partial fractions:

$$\frac{4x}{(2x-1)^2} \equiv \frac{A}{2x-1} + \frac{B}{(2x-1)^2} \equiv \frac{2}{2x-1} + \frac{2}{(2x-1)^2}$$

$$\int \frac{2}{2x-1} + \frac{2}{(2x-1)^2}\,\mathrm{d}x = \ln(2x-1) - \frac{1}{(2x-1)}$$

$$\left[\ln(2x-1) - \frac{1}{2x-1}\right]_3^{\frac{21}{2}} = \ln 20 - \frac{1}{20} - \ln 5 + \frac{1}{5} = \frac{3}{20} + \ln 4$$

Mark as B0M0A0 then:

M1 for $\int \frac{A}{2x-1} + \frac{B}{(2x-1)^2}\,\mathrm{d}x = \alpha \ln(2x-1) + \beta \frac{1}{(2x-1)}$

A1 for $\int \frac{2}{2x-1} + \frac{2}{(2x-1)^2}\,\mathrm{d}x = \ln(2x-1) - \frac{1}{(2x-1)}(+c)$

dM1 Substitutes both correct limits and subtracts either way round. **Depends on first M.**

A1: $\ln 4 + \frac{3}{20}$ or e.g. $\ln 4 + 0.15$ and apply isw if necessary.

or Parts:

$$\int \frac{4x}{(2x-1)^2}\,\mathrm{d}x = -2x(2x-1)^{-1} + \ln(2x-1)$$

$$= \left[-2x(2x-1)^{-1} + \ln(2x-1)\right]_3^{\frac{21}{2}} = -\frac{21}{20} + \ln 20 + \frac{6}{5} - \ln 5 = \frac{3}{20} + \ln 20$$

M1 for $\int \frac{4x}{(2x-1)^2}\,\mathrm{d}x = \alpha x(2x-1)^{-1} + \beta \ln(2x-1)$

A1 for $\int \frac{4x}{(2x-1)^2}\,\mathrm{d}x = -2x(2x-1)^{-1} + \ln(2x-1)(+c)$

dM1 Substitutes both correct limits and subtracts either way round. **Depends on first M.**

A1: $\ln 4 + \frac{3}{20}$ or e.g. $\ln 4 + 0.15$ and apply isw if necessary.

If you find any attempts using a combination of methods that include substitution e.g.

$$\frac{4x}{(2x-1)^2} \equiv \frac{A}{2x-1} + \frac{B}{(2x-1)^2} \equiv \frac{2}{2x-1} + \frac{2}{(2x-1)^2}$$

$$\int \frac{2}{2x-1} + \frac{2}{(2x-1)^2}\,\mathrm{d}x = \left[\ln(2x-1)\right] + \int \frac{2}{u^2}\frac{\mathrm{d}u}{2} \qquad (u = 2x-1)$$

$$\int \frac{2}{2x-1} + \frac{2}{(2x-1)^2}\,\mathrm{d}x = \left[\ln(2x-1)\right]_3^{\frac{21}{2}} + \left[-\frac{1}{u}\right]_5^{20} = \ln 20 - \ln 5 - \frac{1}{20} + \frac{1}{5} = \frac{3}{20} + \ln 4$$

OR

$$\int \frac{4x}{(2x-1)^2}\,\mathrm{d}x = \left[-2x(2x-1)^{-1}\right] + \int \frac{2}{2x-1}\,\mathrm{d}x$$

$$= \left[-2x(2x-1)^{-1}\right] + \int \frac{2}{u}\,\frac{\mathrm{d}u}{2} \qquad (u = 2x-1)$$

$$= \left[-2x(2x-1)^{-1}\right]_3^{\frac{21}{2}} + \left[\ln u\right]_5^{20} = -\frac{21}{20} + \frac{6}{5} + \ln 20 - \ln 5 = \frac{3}{20} + \ln 20$$

if fully correct as above then score full marks otherwise send to review.

Question Number	Scheme	Marks
4(a)	Assume that there exists a positive number k such that $k+\frac{9}{k}<6$	B1
	$k+\frac{9}{k}<6 \Rightarrow k^2+9<6k \Rightarrow k^2-6k+9<0$ **or** $k+\frac{9}{k}<6 \Rightarrow k+\frac{9}{k}-6<0 \Rightarrow \left(\sqrt{k}...\right)\left(\sqrt{k}...\right)<0$ **or** $k+\frac{9}{k}<6 \Rightarrow \left(k+\frac{9}{k}\right)^2<36 \Rightarrow k^2+18+\frac{81}{k^2}-36<0$	M1
	$\Rightarrow \left(k-3\right)^2<0$ **or** $\Rightarrow \left(\sqrt{k}-\frac{3}{\sqrt{k}}\right)^2<0$ **or** $\Rightarrow \left(k-\frac{9}{k}\right)^2<0$	A1
	But numbers squared are $\geqslant 0$, hence $k+\frac{9}{k}\geqslant 6$	A1*
		(4)
(b)	E.g. When $k=-3$, $-3+\frac{9}{-3}\left(=-6\right)$ which is not $\geqslant 6$	B1
		(1)
(b) Alt	$k<0 \Rightarrow \frac{9}{k}<0 \Rightarrow k+\frac{9}{k}<0$ which is not $\geqslant 6$	B1
		(5 marks)

(a)

B1: For setting up the correct contradiction. It must include a word/words such as "assume" or "let" and must be a strict inequality. As a minimum accept just "assume/let $k+\frac{9}{k}<6$".

Condone e.g. Assume that for all positive numbers k, $k+\frac{9}{k}<6$

M1: Starting from $k+\frac{9}{k}<6$ or $k+\frac{9}{k}\leqslant 6$ or $k+\frac{9}{k}=6$, either

- multiplies by k and attempts to reach $k^2...<0$ or $k^2...\leqslant 0$ or $k^2...=0$
- collects terms to one side to reach $k+\frac{9}{k}\pm 6 \leqslant / < / = 0$ and attempts to factorise to $\left(\sqrt{k}...\right)\left(\sqrt{k}...\right)\leqslant / < / = 0$
- squares both sides and collects terms to reach $k^2...<0$ or $k^2...\leqslant 0$ or $k^2...=0$

A1: Reaches $\left(k-3\right)^2...0$ or e.g. $\left(\sqrt{k}-\frac{3}{\sqrt{k}}\right)^2...0$ or e.g. $\left(k-\frac{9}{k}\right)^2...0$ where … is < or $\leqslant$ or =