6. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

The temperature, θ °C, of a car engine, t minutes after the engine is turned off, is modelled by the differential equation

$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = -k(\theta - 15)^2$$

where k is a constant.

Given that the temperature of the car engine
- is 85 °C at the instant the engine is turned off
- is 40 °C exactly 10 minutes after the engine is turned off

(a) solve the differential equation to show that, according to the model

$$\theta = \frac{at + b}{ct + d}$$

where a, b, c and d are integers to be found.

(7)

(b) Hence find, according to the model, the time taken for the temperature of the car engine to reach 20 °C. Give your answer to the nearest minute.

(2)

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Question 6 continued

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Question 6 continued

(Total for Question 6 is 9 marks)

7. Use proof by contradiction to prove that $\sqrt{7}$ is irrational.

(You may assume that if k is an integer and k^2 is a multiple of 7 then k is a multiple of 7)

(4)

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