Question Number	Scheme	Marks
3(a)	$\dfrac{8x-5}{(2x-1)(4x-3)} \equiv \dfrac{A}{2x-1}+\dfrac{B}{4x-3} \Rightarrow 8x-5=A(4x-3)+B(2x-1) \Rightarrow A=...\ B=...$	M1
	$A=1 \quad \text{or} \quad B=2$	A1
	$\dfrac{8x-5}{(2x-1)(4x-3)} \equiv \dfrac{1}{2x-1}+\dfrac{2}{4x-3}$	A1
		(3)
(b)	$\displaystyle\int\left(\frac{1}{2x-1}+\frac{2}{4x-3}\right)\mathrm{d}x=\frac{1}{2}\ln\lvert 2x-1\rvert+\frac{1}{2}\ln\lvert 4x-3\rvert(+c)$ Follow through their A and B	M1, A1ft, A1ft
		(3)
(c)	$\left[\frac{1}{2}\ln(2x-1)+\frac{1}{2}\ln(4x-3)\right]_k^{3k}=\frac{1}{2}\ln(6k-1)(12k-3)-\frac{1}{2}\ln(2k-1)(4k-3)$ $=\frac{1}{2}\ln\dfrac{(6k-1)(12k-3)}{(2k-1)(4k-3)}$	M1
	$\frac{1}{2}\ln\dfrac{(6k-1)(12k-3)}{(2k-1)(4k-3)}=\frac{1}{2}\ln 20 \Rightarrow \dfrac{(6k-1)(12k-3)}{(2k-1)(4k-3)}=20$ $\Rightarrow (6k-1)(12k-3)=20(2k-1)(4k-3) \Rightarrow 88k^2-170k+57=0$	dM1A1
	$88k^2-170k+57=0 \Rightarrow k=...$	ddM1
	$k=\dfrac{3}{2}$	A1
		(5)
		Total 11

(a)

M1: "Correct" P.F form (condoning slips) with strategy to find at least one constant or partial fraction.
Look for an attempt at the correct form (condoning slips) $5-8x=A(4x-3)+B(2x-1)$ followed by values for A or B

A1: One correct fraction or constant

A1: Correct partial fractions which may be awarded for sight of correct PF in part (b).
Award if the correct answer is just written down with little or no working.
Award if the correct partial fractions are seen in (b)

(b)

M1: For $\displaystyle\int\frac{\alpha}{2x-1}\,\mathrm{d}x=\beta\ln\lvert 2x-1\rvert$ **OR** $\displaystyle\int\frac{\alpha}{4x-3}\,\mathrm{d}x=\beta\ln\lvert 4x-3\rvert$

Allow for example brackets instead of moduli, e.g. $\ln(2x-1)\leftrightarrow\ln\lvert 2x-1\rvert$

Condone the omission of moduli/brackets, for example $\displaystyle\int\frac{\alpha}{2x-1}\,\mathrm{d}x=\beta\ln 2x-1$ for this mark only

A1ft: For $\displaystyle\int\frac{A}{2x-1}\,\mathrm{d}x=\frac{A}{2}\ln\lvert 2x-1\rvert$ **OR** $\displaystyle\int\frac{B}{4x-3}\,\mathrm{d}x=\frac{B}{4}\ln\lvert 4x-3\rvert$ Follow through their A and B

Allow for example brackets instead of moduli, e.g . $\ln(2x-1)\leftrightarrow\ln\lvert 2x-1\rvert$

A1ft: For $\int \frac{A}{2x-1}\,\mathrm{d}x + \int \frac{B}{4x-3}\,\mathrm{d}x = \frac{A}{2}\ln|2x-1| + \frac{B}{4}\ln|4x-3|$

or $\int \frac{A}{2x-1}\,\mathrm{d}x + \int \frac{B}{4x-3}\,\mathrm{d}x = \frac{A}{2}\ln(2x-1) + \frac{B}{4}\ln(4x-3)$

Follow through their numerical *A* and *B*. There is no requirement for the + *c*

ISW after a correct answer. This can be awarded (but not implied) from part (c)

Note that $\frac{1}{2}\ln(2x-1)(4x-3)(+c)$ and $\ln\sqrt{(2x-1)(4x-3)}(+c)$ are also correct

(c)

M1: Attempts the use of the correct limits and uses correct log work to combine terms to obtain a single ln term.

It is dependent upon having an answer to part (b) involving two ln terms.

Condone slips/ bracketing errors .

E.g $\left\{\frac{1}{2}\ln(6k-1)+\frac{1}{2}\ln(12k-3)\right\}-\left\{\frac{1}{2}\ln(2k-1)+\frac{1}{2}\ln(4k-3)\right\} = \frac{1}{2}\ln(6k-1)+\frac{1}{2}\ln(12k-3)-\frac{1}{2}\ln(2k-1)+\frac{1}{2}\ln(4k-3)$

$$= \frac{1}{2}\ln\frac{(6k-1)(12k-3)(4k-3)}{(2k-1)}$$

dM1: Eliminates ln's, multiplies up and collects terms to form a polynomial equation in *k*.

It is dependent upon the previous M

A1: Correct 3TQ. The = 0 may be implied by subsequent work

ddM1: Solves 3TQ. Dependent upon both previous M's.

Can be solved via a graphical calculator (you may need to check the answers)

A1: Correct value and no others. Condone $x = \frac{3}{2}$

Solution with limited working in (c)

Case I

$\frac{1}{2}\ln\frac{(6k-1)(12k-3)}{(2k-1)(4k-3)} = \frac{1}{2}\ln 20 \Rightarrow x$ or $k = \frac{3}{2}$ SC 11000

Case II

$\frac{(6k-1)(12k-3)}{(2k-1)(4k-3)} = 20 \Rightarrow x$ or $k = \frac{3}{2}$ SC 11100

Question Number	Scheme	Marks
4(a)	Attempts direction vector by subtracting $(5\mathbf{i}+6\mathbf{j}+3\mathbf{k})$ and $(4\mathbf{i}+8\mathbf{j}+\mathbf{k})$ either way around.	M1
	E.g. $\mathbf{r}=4\mathbf{i}+8\mathbf{j}+\mathbf{k}+\lambda(\mathbf{i}-2\mathbf{j}+2\mathbf{k})$, $\mathbf{r}=5\mathbf{i}+6\mathbf{j}+3\mathbf{k}+\mu(\mathbf{i}-2\mathbf{j}+2\mathbf{k})$	A1
		(2)
(b)	E.g $\overrightarrow{PC}=\begin{pmatrix}4+\lambda\\8-2\lambda\\1+2\lambda\end{pmatrix}-\begin{pmatrix}2\\-2\\1\end{pmatrix}=\begin{pmatrix}2+\lambda\\10-2\lambda\\2\lambda\end{pmatrix}$	M1
	Uses $\overrightarrow{PC}.(\mathbf{i}-2\mathbf{j}+2\mathbf{k})=0$ E.g. $\Rightarrow 1(2+\lambda)-2(10-2\lambda)+2\times 2\lambda=0\Rightarrow\lambda=...$	dM1
	E.g I $\lambda=2\Rightarrow\mathbf{c}=(4+2)\mathbf{i}+(8-4)\mathbf{j}+(1+4)\mathbf{k}$ E.g II $\mu=1\Rightarrow\mathbf{c}=(5+1)\mathbf{i}+(6-2)\mathbf{j}+(3+2)\mathbf{k}$	ddM1
	$(6,\ 4,\ 5)$	A1
		(4)
(c)	$\overrightarrow{OP'}=\mathbf{p}+2\overrightarrow{PC}=2\mathbf{i}-2\mathbf{j}+\mathbf{k}+2(4\mathbf{i}+6\mathbf{j}+4\mathbf{k})$	M1
	$(10,\ 10,\ 9)$	A1
		(2)
(d)	$\left\lvert\overrightarrow{PP'}\right\rvert=2\left\lvert\overrightarrow{PC}\right\rvert=2\sqrt{4^2+6^2+4^2}=...$	M1
	$4\sqrt{17}$	A1
		(2)
		Total 10

General rule in this question: If no method is shown look for two "correct" components for their vector

(a) **There are many correct versions so please check the candidates answer carefully**

M1: Attempts the direction of *l* by subtracting $(5\mathbf{i}+6\mathbf{j}+3\mathbf{k})$ and $(4\mathbf{i}+8\mathbf{j}+\mathbf{k})$ either way around.

If no method is shown look for two correct components of $(\pm 1\mathbf{i}\pm 2\mathbf{j}\pm 2\mathbf{k})$ or a multiple of this

A1: Any correct equation including the lhs of '$\mathbf{r}=$'

Allow in the form $\mathbf{r}=\begin{pmatrix}4\\8\\1\end{pmatrix}+\lambda\begin{pmatrix}1\\-2\\2\end{pmatrix}$ but $l=\begin{pmatrix}4\\8\\1\end{pmatrix}+\lambda\begin{pmatrix}1\\-2\\2\end{pmatrix}$ and is $\mathbf{r}=\begin{pmatrix}4\\8\\1\end{pmatrix}+\lambda\begin{pmatrix}\mathrm{i}\\-2\mathrm{j}\\2\mathrm{k}\end{pmatrix}$ A0

(b)

M1: Attempts the general vector from *P* to *l* forming $\overrightarrow{PC}$ where *C* is a general point on *l*.

Look for an attempt to subtract their "$\begin{pmatrix}4+\lambda\\8-2\lambda\\1+2\lambda\end{pmatrix}$" and $\begin{pmatrix}2\\-2\\1\end{pmatrix}$ either way around

dM1: Attempts $\overrightarrow{PC}.\text{their}(\mathbf{i}-2\mathbf{j}+2\mathbf{k})=0$ and solves for λ.

It is dependent upon the previous M and the scalar product of this vector must be attempted with their gradient for the line *l* to produce and solve a linear equation for λ.

ddM1: Uses their λ to find *C*. Scored for substituting their correctly found λ into their equation for *l*

A1: Correct coordinates or vector.

Only penalise an error such as $\begin{pmatrix}6\mathrm{i}\\4\mathrm{j}\\5\,\mathrm{k}\end{pmatrix}$ once, the first time that it is made. Hence award if a mark has already been withheld for such a mistake.