Question Number	Scheme	Marks
8 (a)	At $t=\frac{\pi}{4}$ $P=\left(\frac{1}{2},2\right)$	**B1**
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y/\mathrm{d}t}{\mathrm{d}x/\mathrm{d}t}=\frac{2\sec^2 t}{2\sin t\cos t}=4$ when $t=\frac{\pi}{4}$	**M1 A1**
	Equation of l: $y-2=-\frac{1}{4}\left(x-\frac{1}{2}\right)\Rightarrow 8y-16=-2x+1\Rightarrow 8y+2x=17*$	**dM1 A1 * cso**
		(5)
(b)	$\int y\frac{\mathrm{d}x}{\mathrm{d}t}\,\mathrm{d}t=\int 2\tan t\times 2\sin t\cos t\,\mathrm{d}t$	**M1**
	$=\int 4\sin^2 t\,\mathrm{d}t$	**A1**
	$=\int 2-2\cos 2t\,\mathrm{d}t\ =\ 2t-\sin 2t$	**dM1 A1**
	Total area of $S=\left[2t-\sin 2t\right]_0^{\frac{\pi}{4}}+\frac{1}{2}\times 8\times 2=\frac{\pi}{2}-1+8=\frac{\pi}{2}+7$	**M1 A1**
		(6)
		(11 marks)

Notes:

(a)

B1: Correct coordinates for P stated or implied by working.

M1: Attempts to find $\frac{\mathrm{d}y}{\mathrm{d}x}$ using $\frac{\mathrm{d}y/\mathrm{d}t}{\mathrm{d}x/\mathrm{d}t}$ at $t=\frac{\pi}{4}$. Condone poor differentiation. Substitution of the $\frac{\pi}{4}$ is sufficient for the method. Alternatively, may attempt $\frac{\mathrm{d}x}{\mathrm{d}y}$ or $-\frac{\mathrm{d}x}{\mathrm{d}y}$. Accept a value following finding $\frac{\mathrm{d}y}{\mathrm{d}x}$ (or its reciprocal etc) as an attempt to evaluate at $t=\frac{\pi}{4}$ if no contrary working is shown **but check carefully** as the correct answer may arise from incorrect working.

A1: Correct $\frac{\mathrm{d}y}{\mathrm{d}x}=4$ (oe equation) following correct differentiation. May be implied.

dM1: Attempts to find the equation of the normal at $t=\frac{\pi}{4}$. It is dependent upon the previous M and use of their P. The value of the gradient used must be correct for their differential.

A1*: **cso** Correct proof leading to $8y+2x=17$

(b)

M1: Attempts $\int y\frac{\mathrm{d}x}{\mathrm{d}t}\,\mathrm{d}t=\int 2\tan t\times"2\sin t\cos t"\,\mathrm{d}t$ with their $\frac{\mathrm{d}x}{\mathrm{d}t}$ condoning slips on coefficients.

A1: $\int 4\sin^2 t\,\mathrm{d}t$

dM1: Uses $\cos 2t = \pm 1 \pm 2\sin^2 t$ and integrates $\int \pm p \pm q\cos 2t \, \mathrm{d}t$ to a form $\pm at \pm b\sin 2t$

See note below.

A1: $\int y \frac{\mathrm{d}x}{\mathrm{d}t} \, \mathrm{d}t = 2t - \sin 2t$ See note below.

M1: Full method to find area of region *S*. Finds the sum of their values for $\int_0^{\frac{\pi}{4}} y \frac{\mathrm{d}x}{\mathrm{d}t} \mathrm{d}t$ and $\frac{1}{2}\left(\frac{17}{2} - P_x\right) \times P_y$. Condone poor integration for this mark as long as they are attempting to apply the correct limits to their result. They may attempt the area under the line by integration:

$\int_{P_x}^{\frac{17}{2}} -\frac{1}{4}x + \frac{17}{8} \mathrm{d}x$ In such a method condone minor slips, but must be attempting correct limits.

A1: $\frac{\pi}{2} + 7$

Note: If the *t*'s becomes *x*'s during the integration, then allow the M's and the A's if recovered but if $2x - \sin 2x$ or with mixed variables is found and *x* values substituted then it is M1A0 for the integral and M0 for the method for area.

8 (a) Alt	At $t = \frac{\pi}{4}$ $P = \left(\frac{1}{2}, 2\right)$	**B1**
	$y = \frac{2\sin t}{\cos t} = \frac{2\sqrt{x}}{\sqrt{1-x}}$ $y^2 = \frac{4x}{1-x}$ $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x^{-\frac{1}{2}}(1-x)^{\frac{1}{2}} - 2\sqrt{x} \times -\frac{1}{2}(1-x)^{-\frac{1}{2}}}{1-x} \Rightarrow \left.\frac{\mathrm{d}y}{\mathrm{d}x}\right\|_{x=\frac{1}{2}} = 4$ or $2y\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4(1-x) - 4x \times -1}{(1-x)^2} \Rightarrow \left.\frac{\mathrm{d}y}{\mathrm{d}x}\right\|_{x=\frac{1}{2}, y=2} = 4$ oe	**M1 A1**
	Equation of *l*: $y - 2 = -\frac{1}{4}\left(x - \frac{1}{2}\right) \Rightarrow 8y - 16 = -2x + 1 \Rightarrow 8y + 2x = 17$ *	**dM1 A1 * cso**
		(5)

(a)

B1: Correct coordinate for *P* $\left(\frac{1}{2}, 2\right)$ stated or implied by working.

M1: Attempts to find Cartesian equation for *C*, any form, and attempts $\frac{\mathrm{d}y}{\mathrm{d}x}$ (or an equivalent as main scheme) with appropriate differentiation methods for their Cartesian form, allowing for slips and finds *x* and/or *y* using $t = \frac{\pi}{4}$ and evaluate the derivative with these values.

A1: Correct $\frac{\mathrm{d}y}{\mathrm{d}x} = 4$ (oe equation) following correct differentiation and from correct work.

dM1: Attempts to find the equation of the normal at their x and y values.
It is dependent upon the previous M and use of their P.

A1*: **cso** Correct proof leading to $8y + 2x = 17$

Question Number	Scheme	Marks
9 (a)	Let $p=3k+2$ then $\left(3k+2\right)^3=27k^3+54k^2+36k+8$	**M1**
	$=3\times\left(9k^3+18k^2+12k+3\right)-1$ not a multiple of 3	**A1**
	So p cannot be of form $3k + 1$ or $3k + 2$, since p^3 is a multiple of 3. Hence p must be a multiple of 3, a contradiction of our assumption, hence for all integers p, when p^3 is a multiple of 3, then p is a multiple of 3	**A1**
		(3)
(b)	Assumption: there exist (integers) p and q such that $\sqrt[3]{3}=\frac{p}{q}$ (where p and q have no (non-trivial) common factors.)	**B1**
	Then $\sqrt[3]{3}=\frac{p}{q}\Rightarrow p^3=3q^3$	**M1**
	So p^3 is a multiple of 3 and (so) p is a multiple of 3	**A1**
	But $p=3k\Rightarrow 27k^3=3q^3\Rightarrow q^3=9k^3$	**dM1**
	Hence q^3 is a multiple of 3 so q is a multiple of 3, but as p and q have no (non-trivial) common factors, this is a contradiction. Hence $\sqrt[3]{3}$ is an irrational number.*	**A1***
		(5)
		(8 marks)

Notes:

(a)

M1: Attempts to expand $\left(3k+2\right)^3$ or $\left(3k-1\right)^3$.

Look for a cubic expression with **4 terms** with at least **two correct** (allowing for incorrect signs).

A1: Achieves a correct $3\times(...)+r,|r|<10$ form for the expansion and states not a multiple of 3.

Suitable forms include $\left(3k+2\right)^3=3\left(9k^3+18k^2+12k+2\right)+2=3\left(9k^3+18k^2+12k+3\right)-1$ or

$3\left(9k^3+18k^2+12k\right)+8$ or $\left(3k-1\right)^3=3\left(9k^3-9k^2+3k\right)-1$ etc.

Alternatively, achieves correct $\left(3k+2\right)^3=27k^3+54k^2+36k+8$ or $\left(3k-1\right)^3=27k^3-27k^2+9k-1$ with a **reason** why it is not a multiple of 3 e.g 3 divides 27, 54 and 36, but not 8 hence not divisible by 3.

A1: Completes the proof. Must have scored both previous marks and a reference to **both cases** (in some form) leading to a "**contradiction"** and some indication that **proof is complete**. It is unlikely to be as complete as that shown in the scheme, but all three bold points must be conveyed. E.g. as a minimum after satisfying the first A "both cases give a contradiction hence the original statement is true".

(b)

May use different letters throughout.