**7.** The volume $V\,\text{cm}^3$ of a spherical balloon with radius $r\,\text{cm}$ is given by the formula

$$V = \frac{4}{3}\pi r^3$$

(a) Find $\dfrac{\mathrm{d}V}{\mathrm{d}r}$ giving your answer in simplest form.

**(1)**

At time $t$ seconds, the volume of the balloon is increasing according to the differential equation

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{900}{(2t+3)^2} \qquad t \geqslant 0$$

Given that $V = 0$ when $t = 0$

(b) (i) solve this differential equation to show that

$$V = \frac{300t}{2t+3}$$

(ii) Hence find the upper limit to the volume of the balloon.

**(5)**

(c) Find the radius of the balloon at $t = 3$, giving your answer in cm to 3 significant figures.

**(3)**

(d) Find the rate of increase of the radius of the balloon at $t = 3$, giving your answer to 2 significant figures. Show your working and state the units of your answer.

**(3)**

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

**Question 7 continued**

**Question 7 continued**

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

**Question 7 continued**

**(Total for Question 7 is 12 marks)**

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

**8.**

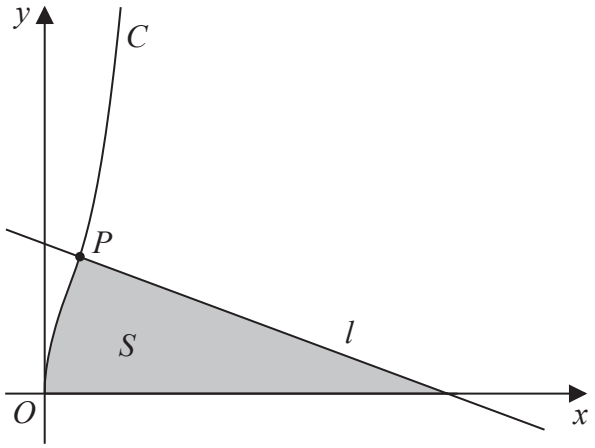



**Figure 3**

**In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

A curve $C$ has parametric equations

$$x = \sin^2 t \qquad y = 2\tan t \qquad 0 \leqslant t < \frac{\pi}{2}$$

The point $P$ with parameter $t = \frac{\pi}{4}$ lies on $C$.

The line $l$ is the normal to $C$ at $P$, as shown in Figure 3.

(a) Show, using calculus, that an equation for $l$ is

$$8y + 2x = 17$$

**(5)**

The region $S$, shown shaded in Figure 3, is bounded by $C$, $l$ and the $x$-axis.

(b) Find, using calculus, the exact area of $S$.

**(6)**

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA