Question Number	Scheme	Marks
3.	States or implies Volume = $\pi\int_{\sqrt{5}}^{5}\left(\sqrt{\frac{3x}{3x^2+5}}\right)^2 \mathrm{d}x$	**B1**
	$\int\left(\sqrt{\frac{3x}{3x^2+5}}\right)^2 \mathrm{d}x = \int\frac{3x}{(3x^2+5)}\mathrm{d}x = \frac{1}{2}\ln(3x^2+5)$	**M1A1**
	Volume = $\{\pi\}\left(\frac{1}{2}\ln(3\times25+5)-\frac{1}{2}\ln(3\times5+5)\right)$	**M1**
	$=\pi\ln 2$	**A1**
		(5 marks)

Notes:

B1: States or implies Volume = $\pi\int_{\sqrt{5}}^{5}\left(\sqrt{\frac{3x}{3x^2+5}}\right)^2 \mathrm{d}x$ o.e. The limits may be implied by subsequent work, and the dx may be missing. This is for knowing the correct formula rather than for notation. The π may be implied by later work.

M1: Attempts $\int\left(\sqrt{\frac{3x}{3x^2+5}}\right)^2 \mathrm{d}x$ to achieve $k\ln(3x^2+5)$ (oe). May use substitution, either $u = 3x^2$ or $u = 3x^2+5$, in which case they should achieve $k\ln(u+5)$ or $k\ln(u)$ (oe) respectively. Allow if the brackets are missing.

A1: Correct result of integration, which may be left unsimplified. May be in terms of u if a substitution has been used. Allow if missing brackets are recovered, but A0 if never recovered.

M1: Having achieved an integral of the form $p(x)\ln(3x^2+5)$ (allowing for missing brackets) where $p(x)$ is constant or a polynomial in x (oe in terms of u for substitution), uses the limits within their integral - substitutes **correct limits for their variable** and subtracts, allowing either way round. The π may be missing for this mark.

A1: $\pi\ln 2$ cao. Note $\frac{\pi}{2}\ln 4$ is A0 as form is not as specified.

Question Number	Scheme	Marks
4 (a)	$a=3, b=5$	**B1**
	E.g $u=\sqrt{2x+1} \Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}u}=u$ or $\dfrac{\mathrm{d}u}{\mathrm{d}x}=(2x+1)^{-\frac{1}{2}}$ o.e.	**B1**
	$\int \sqrt{8x+4}\, \mathrm{e}^{\sqrt{2x+1}}\, \mathrm{d}x = \int 2u\, \mathrm{e}^{u}\, u\, \mathrm{d}u$	**M1**
	$= \int_3^5 2u^2\, \mathrm{e}^{u}\, \mathrm{d}u$	**A1**
		(4)
(b)	$\int \sqrt{8x+4}\, \mathrm{e}^{2x+1}\, \mathrm{d}x = \int 2u^2\, \mathrm{e}^{u}\, \mathrm{d}u$	
	$= 2u^2\mathrm{e}^{u} - \int 4u\, \mathrm{e}^{u}\, \mathrm{d}u$	**M1**
	$= = 2u^2\mathrm{e}^{u} - \left(4u\mathrm{e}^{u} - 4\mathrm{e}^{u}\right) = 2u^2\mathrm{e}^{u} - 4u\mathrm{e}^{u} + 4\mathrm{e}^{u}$	**dM1 A1ft**
	$\int_4^{12} \sqrt{8x+4}\, \mathrm{e}^{\sqrt{2x+1}}\, \mathrm{d}x = \left[2u^2\mathrm{e}^{u} - 4u\mathrm{e}^{u} + 4\mathrm{e}^{u}\right]_3^5 = \left(50\mathrm{e}^{5} - 20\mathrm{e}^{5} + 4\mathrm{e}^{5}\right) - \left(18\mathrm{e}^{3} - 12\mathrm{e}^{3} + 4\mathrm{e}^{3}\right)$	**ddM1**
	$= 34\mathrm{e}^{5} - 10\mathrm{e}^{3}$	**A1**
		(5)
		(9 marks)

Notes:

(a)

B1: For both $a=3, b=5$ seen in their solution. Allow if these are recovered in (b).

B1: For a correct expression involving $\dfrac{\mathrm{d}u}{\mathrm{d}x}$ or $\dfrac{\mathrm{d}x}{\mathrm{d}u}$ or du and dx separately. May be unsimplified

M1: Attempts to fully change $\int \sqrt{8x+4}\, \mathrm{e}^{\sqrt{2x+1}}\, \mathrm{d}x$ into an integral with respect to u. Must include an attempt at replacing dx to get du so M0 if there are no d terms present or dx becomes du without an attempt at connecting them first (ie there must have been an attempt at $\dfrac{\mathrm{d}u}{\mathrm{d}x}$ oe).

A1: Complete method to show I = $\int_3^5 2u^2\, \mathrm{e}^{u}\, \mathrm{d}u$. Must include the correct limits and the du.

(b) Note: you may see different ways of presenting the application of parts e.g D/I method.

M1: Use of integration by parts once to obtain $pu^2\mathrm{e}^{u} - \int qu\, \mathrm{e}^{u}\, \mathrm{d}u$, where $p, q > 0$ (if k is positive, otherwise signs will be opposite) and may be in terms of k (as can the dM mark).

dM1: Completely integrates by parts twice to a form $pu^2\mathrm{e}^{u} - qu\mathrm{e}^{u} \pm r\mathrm{e}^{u}$ where $p, q > 0$ (if $k > 0$ as before). Note they may evaluate in stages, but look for the complete integration overall.

A1ft: $\int_a^b ku^2\, \mathrm{e}^{u}\, \mathrm{d}u = ku^2\mathrm{e}^{u} - 2ku\mathrm{e}^{u} + 2k\mathrm{e}^{u}$ (oe) accepted with k or their value for k. May have the last two terms bracketed but must be seen as a complete answer in their work.

ddM1: Substitutes their a and b into a form $pu^2\mathrm{e}^{u} - qu\mathrm{e}^{u} \pm r\mathrm{e}^{u}$ and subtracts (either way), but must be using a value for k at this stage. May be done in stages.