8. A student was asked to prove by contradiction that

"there are no positive integers x and y such that $3x^2 + 2xy - y^2 = 25$"

The start of the student's proof is shown in the box below.

Assume that integers x and y exist such that $3x^2 + 2xy - y^2 = 25$

$$\Rightarrow (3x - y)(x + y) = 25$$

If $(3x - y) = 1$ and $(x + y) = 25$

$$\left.\begin{aligned} 3x - y &= 1 \\ x + y &= 25 \end{aligned}\right\} \Rightarrow 4x = 26 \Rightarrow x = 6.5, y = 18.5$$ Not integers

Show the calculations and statements that are needed to complete the proof.

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Question 8 continued

(Total for Question 8 is 4 marks)

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9. With respect to a fixed origin O, the equations of lines l_1 and l_2 are given by

$$l_1 : \mathbf{r} = \begin{pmatrix} 2 \\ 8 \\ 10 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}$$

$$l_2 : \mathbf{r} = \begin{pmatrix} -4 \\ -1 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ 4 \\ 8 \end{pmatrix}$$

where λ and μ are scalar parameters.

Prove that lines l_1 and l_2 are skew.

(5)

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