Question Number	Scheme	Marks
4(a)	$\dfrac{1}{\sqrt{4-x^2}}=\left(4-x^2\right)^{-\frac{1}{2}}=4^{-\frac{1}{2}}\left(1-...\right)$	B1
	$\left(1-\frac{1}{4}x^2\right)^{-\frac{1}{2}}=1+\left(-\frac{1}{2}\right)\left(-\frac{1}{4}x^2\right)+\dfrac{\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)}{2}\left(-\frac{1}{4}x^2\right)^2+\dfrac{\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)\times\left(-\frac{5}{2}\right)}{3!}\left(-\frac{1}{4}x^2\right)^3$	M1, A1
	$\dfrac{1}{\sqrt{4-x^2}}=\dfrac{1}{2}+\dfrac{1}{16}x^2+\dfrac{3}{256}x^4+\dfrac{5}{2048}x^6$	A1, A1
		(5)
(b)	$\left\|x\right\|<2$	B1
		(1)
(c)	Substitutes an appropriate value of x in both sides with LHS in terms of $\sqrt{3}$	M1
	E.g. with $x=1$ $\sqrt{3}=\dfrac{2048}{1181}$ or $\dfrac{3543}{2048}$	A1
		(2)
		(8 marks)

(a) Note that the first mark in (a) is M1 on epen. We are now marking it as B1

B1: Correct constant term/ factor.

Accept as $4^{-\frac{1}{2}}$ or $\dfrac{1}{2}$ as the constant term or as the factor with the bracket starting (1....

M1: Correct attempt at the third or the fourth term of binomial expansion of form $\left(1+ax^2\right)^{-\frac{1}{2}}$

Look for a correct binomial coefficient with a correct power of x

E.g. $\dfrac{\left(-\frac{1}{2}\right)\times\left(-\frac{1}{2}-1\right)}{2}\left(\pm ax^2\right)^2$ or $\dfrac{\left(-\frac{1}{2}\right)\times\left(-\frac{1}{2}-1\right)\times\left(-\frac{1}{2}-2\right)}{3!}\left(\pm ax^2\right)^3$ where a could be 1

A1: Correct unsimplified

For example

$\left(1-\frac{1}{4}x^2\right)^{-\frac{1}{2}}=1+\left(-\frac{1}{2}\right)\left(-\frac{1}{4}x^2\right)+\dfrac{\left(-\frac{1}{2}\right)\times\left(-\frac{1}{2}-1\right)}{2}\left(-\frac{1}{4}x^2\right)^2+\dfrac{\left(-\frac{1}{2}\right)\times\left(-\frac{1}{2}-1\right)\times\left(-\frac{1}{2}-2\right)}{3!}\left(-\frac{1}{4}x^2\right)^3$

A1: Two correct and simplified terms of $\dfrac{1}{2}+\dfrac{1}{16}x^2+\dfrac{3}{256}x^4+\dfrac{5}{2048}x^6$

A1: $\dfrac{1}{2}+\dfrac{1}{16}x^2+\dfrac{3}{256}x^4+\dfrac{5}{2048}x^6$. Accept as a list, ignore any terms with indices greater than these.

(b)

B1: States a correct range. E.g $\left|x\right|<2$ or $-2<x<2$ o.e.

(c) For example: See **

M1: Substitutes an exact value of x into both sides of their expansion that enables $\sqrt{3}$ to be found

.

E.g. $x=1$ $\dfrac{1}{\sqrt{4-1^2}}=\dfrac{1}{2}+\dfrac{1}{16}\times1^2+\dfrac{3}{256}\times1^4+\dfrac{5}{2048}\times1^6$

Note that $x=-1$ and $x^2=1$ all work with the same results

Alternatively correctly solves $\dfrac{1}{\sqrt{4-x^2}}=\sqrt{3}$ o.e. and substitutes this into their expansion

A1: With $x=1$ $\sqrt{3}=\dfrac{2048}{1181}$ or $\dfrac{3543}{2048}$. This can only be scored from a correct expansion

**There are lots of values of *x* that work, all more difficult than the one above.

E.g $x^2=\dfrac{8}{3}\Rightarrow\dfrac{1}{\sqrt{4-x^2}}=\dfrac{\sqrt{3}}{2}$ and $x^2=\dfrac{11}{3}\Rightarrow\dfrac{1}{\sqrt{4-x^2}}=\sqrt{3}$

In both these cases the fractions become much more difficult to find.

For $x^2=\dfrac{8}{3}\Rightarrow\sqrt{3}=\dfrac{43}{27}$ and for $x^2=\dfrac{11}{3}\Rightarrow\sqrt{3}=\dfrac{55687}{55296}$

Alt I (a) By direct expansion

$$\frac{1}{\sqrt{4-x^2}}=\left(4-x^2\right)^{-\frac{1}{2}}=4^{-\frac{1}{2}}+\left(-\frac{1}{2}\right)\times4^{-\frac{3}{2}}\left(-x^2\right)^1+\frac{\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)}{2}\times4^{-\frac{5}{2}}\left(-x^2\right)^2+\frac{\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)\times\left(-\frac{5}{2}\right)}{3!}\times4^{-\frac{7}{2}}\left(-x^2\right)^3$$

B1: For $4^{-\frac{1}{2}}+$

M1: A correct attempt at the third or fourth terms condoning sign slips on the $-x^2$

A1: Correct and unsimplified expansion. See above

A1: Two correct and simplified terms of $\dfrac{1}{2}+\dfrac{1}{16}x^2+\dfrac{3}{256}x^4+\dfrac{5}{2048}x^6$

A1: $\dfrac{1}{2}+\dfrac{1}{16}x^2+\dfrac{3}{256}x^4+\dfrac{5}{2048}x^6$. Accept as a list

Alt II (a) By difference of two squares

$$\frac{1}{\sqrt{4-x^2}}=\left(2-x\right)^{-\frac{1}{2}}\times\left(2+x\right)^{-\frac{1}{2}}=4^{-\frac{1}{2}}+.....$$

B1: For $4^{-\frac{1}{2}}+$

M1: For correctly writing $\dfrac{1}{\sqrt{4-x^2}}=\left(2-x\right)^{-\frac{1}{2}}\times\left(2+x\right)^{-\frac{1}{2}}=...\left(1-\dfrac{x}{2}\right)^{-\frac{1}{2}}\times\left(1+\dfrac{x}{2}\right)^{-\frac{1}{2}}=$

with a correct attempt at the third of fourth term in either expansion, followed by an attempt to combine both the expansions.

A1: One correct term in x^2, x^4 or x^6

A1: Two correct and simplified terms of $\dfrac{1}{2}+\dfrac{1}{16}x^2+\dfrac{3}{256}x^4+\dfrac{5}{2048}x^6$ with no terms in x, x^3 or x^5

A1: Fully correct

Question Number	Scheme	Marks
5.	States or implies Volume = $\displaystyle\int_{\frac{1}{\sqrt{2}}}^{k}\pi\left(\frac{12\sqrt{x}}{(2x^2+3)^{1.5}}\right)^2\mathrm{d}x$	B1
	Attempts $\displaystyle\int\left(\frac{12\sqrt{x}}{(2x^2+3)^{1.5}}\right)^2\mathrm{d}x=\int\frac{144x}{(2x^2+3)^3}\mathrm{d}x=-18(2x^2+3)^{-2}$	M1A1
	Sets $\quad -\dfrac{18\pi}{(2k^2+3)^2}+\dfrac{18\pi}{(2\times\frac{1}{2}+3)^2}=\dfrac{713}{648}\pi$	M1
	$\Rightarrow -\dfrac{18}{(2k^2+3)^2}+\dfrac{9}{8}=\dfrac{713}{648}\Rightarrow(2k^2+3)^2=729\Rightarrow k^2=\ldots$	ddM1
	$k=2\sqrt{3}$	A1
		(6 marks)

B1: States or implies Volume = $\displaystyle\int_{\frac{1}{\sqrt{2}}}^{k}\pi\left(\frac{12\sqrt{x}}{(2x^2+3)^{1.5}}\right)^2\mathrm{d}x$ o.e. including limits and π.

You can condone a missing dx and the limits the wrong way around.

You may not see the limits until the integration has been attempted which is fine.

M1: Attempts $\displaystyle\int\left(\frac{\sqrt{x}}{(2x^2+3)^{1.5}}\right)^2\mathrm{d}x$ to achieve $A(2x^2+3)^{-2}$ o.e.

With substitution of $u=2x^2+3$ look for Au^{-2}

A1: Correct integration of $\displaystyle\int\left(\frac{12\sqrt{x}}{(2x^2+3)^{1.5}}\right)^2\mathrm{d}x$ to $-18(2x^2+3)^{-2}$ or $-18u^{-2}$ which may be left unsimplified.

Note that this can be scored with or without the π and may even have an incorrect coefficient of 2π

M1: Uses the limits (subtracts either way around) within their attempted integration of y^2 NOT y and sets equal to $\frac{713}{648}\pi$. Condone poor attempts at integrating the y^2 here.

Note that if they change limits to u where $u=2x^2+3$ look for limits of $2k^2+3$ and 4

ddM1: Dependent upon both previous M's. It is for proceeding to

either $(2k^2+3)^{\pm2}=B$ where $B>0$ leading to a value for k^2 or k

or setting up a quadratic equation in k^2 leading to a value for k^2 or k

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FYI

$$k^4+3k^2-180=0\Rightarrow\left(k^2-12\right)\left(k^2+15\right)=0\Rightarrow k^2=12$$

A1: $k=2\sqrt{3}$ or exact equivalent such as $\sqrt{12}$ Condone if $k=\pm2\sqrt{3}$ o.e.