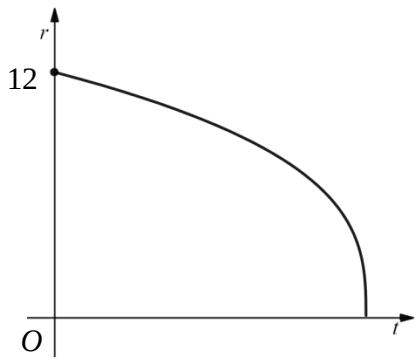
Question Number	Scheme	Marks
10 (a)	States or uses $\frac{\mathrm{d}r}{\mathrm{d}t} \propto \pm\frac{1}{r^2}$ or $\frac{\mathrm{d}r}{\mathrm{d}t} = \pm\frac{k}{r^2}$	B1
	$\frac{\mathrm{d}r}{\mathrm{d}t} = \pm\frac{k}{r^2} \Rightarrow \int r^2 \mathrm{d}r = \int \pm k \mathrm{d}t$	
	$\frac{1}{3}r^3 = \pm kt + c$	M1 A1
	Substitutes $t = 0, r = 12 \Rightarrow c = \left(576\right)$	
	AND $t = 15, r = 6$ and their $c = \left(576\right) \Rightarrow k = \left(\pm 33.6\right)$	M1
	$\frac{1}{3}r^3 = -33.6t + 576$ or $r^3 = -100.8t + 1728$ o.e.	A1
		(5)
(b)	Sets $r = 0 \Rightarrow -100.8t + 1728 = 0$	M1
	$\frac{120}{7}$ minutes or awrt 17.1 minutes	A1
		(2)
(c)		B1
		(1)
		(8 marks)

(a)

B1: States or uses $\frac{\mathrm{d}r}{\mathrm{d}t} \propto \pm\frac{1}{r^2}$, $\frac{\mathrm{d}r}{\mathrm{d}t} = \pm\frac{k}{r^2}$ or $\frac{\mathrm{d}r}{\mathrm{d}t} = \pm\frac{1}{kr^2}$ Accept $k \leftrightarrow \lambda$ or other constant. Condone $\frac{\mathrm{d}r}{\mathrm{d}t} = \pm\frac{1}{r^2}$

M1: Integrates to achieve an expression of the correct form $...r^3 = ...t\left(+c\right)$ o.e

There is no requirement for + c

A1: Correct integration with two different unknown constants. E.g. look for $\frac{1}{3}r^3 = \pm kt + c$ o.e

So if for example $\frac{\mathrm{d}r}{\mathrm{d}t} = -\frac{1}{r^2} \Rightarrow \frac{1}{3}r^3 = -t + c$ the score would be B1, M1, A0 then dM0, A0

M1: Uses both boundary conditions to form an equation involving r and t. For this to be awarded there must be two different unknown constants that are initially correctly placed.

It is dependent upon having achieved $...r^n = \pm"k"t + "c"$ which may have been achieved from an

incorrect assumption. E.g. $\frac{\mathrm{d}r}{\mathrm{d}t} = \pm kr^2$

A1: Correct equation. E.g. $\frac{1}{3}r^3 = -33.6t + 576$ or $r^3 = -100.8t + 1728$ o.e. such as $t = -\frac{5}{504}r^3 + \frac{120}{7}$

(b)

M1: Sets $r = 0 \Rightarrow -100.8t + 1728 = 0 \Rightarrow t = ...$ Follow through on their equation. Condone if this produces a negative value for *t*. Alt sets $V = 0$ to find $t = ...$

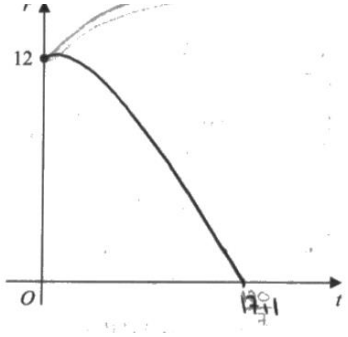
A1: Awrt 17.1 minutes. Must include the units. Also allow 17 minutes 8 seconds or 17 minutes 9 seconds

(c)

B1: For the correct shaped curve only in quadrant 1 starting at (0,12), ignoring value of *t*.

As $\frac{dr}{dt} = -\frac{k}{r^2}$ the gradient should appear to get increasingly steeper.

If there are two curves given on the axes, both need to be correct for this mark to be awarded
Do not be concerned if it is not infinite at the *t* axis.



The curve on the left would be at the limit of what we would allow
as the gradient on the left hand side does appear to increase.
We condone its appearance as it approaches the *t*- axis.
It does not get less steep

.....
..

Note that it is possible to use $V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dr} = 4\pi r^2$ with $\frac{dr}{dt} = \pm\frac{k}{r^2}$ to set up and solve

$\frac{dV}{dt} = \pm 4\pi k$ or $\frac{dV}{dt} = \pm\beta$ In this case the equation becomes $V = 2304\pi - \left(\frac{672\pi}{5}\right)t$

(a)

B1: States or uses either $\frac{dV}{dt} = \pm\beta$ or its exact value which is $\frac{dV}{dt} = -\frac{2016\pi}{15}$ o.e such as $\frac{dV}{dt} = -\frac{672\pi}{5}$.

M1: For $\frac{dV}{dt} = \pm\beta \Rightarrow V = \pm\beta t + c$ and attempts to find the values of β and *c* using the given conditions.

It is dependent upon knowing that $V = \alpha r^3$ and using this to find *V* at *r* = 6 and 12 with *t* = 15 and *t* = 0

A1: $V = 2304\pi - \left(\frac{672\pi}{5}\right)t$

M1: Substitutes $V = \alpha r^3$ to form an equation linking r^3 with *t*.
It is dependent on the previous M1 in this method

A1: Achieves $\frac{4}{3}\pi r^3 = 2304\pi - \left(\frac{672\pi}{5}\right)t$

(b)

M1: Sets $V = 0$ o.e and finds *t*

Question Number	Scheme	Marks
11 (a)	$(x+y)^3+10y^2=108x$	
	Differentiates $10y^2$ to $20y\dfrac{dy}{dx}$	B1
	Differentiates $(x+y)^3$ to $3(x+y)^2\left(1+\dfrac{dy}{dx}\right)$	M1
	$3(x+y)^2\left(1+\dfrac{dy}{dx}\right)+20y\dfrac{dy}{dx}=108$	A1
	$3(x+y)^2\dfrac{dy}{dx}+20y\dfrac{dy}{dx}=108-3(x+y)^2$	dM1
	$\dfrac{dy}{dx}=\dfrac{108-3(x+y)^2}{20y+3(x+y)^2}$ *	A1*
		(5)
(b)	Deduces that P and Q are where $108-3(x+y)^2=0\Rightarrow(x+y)^2=36$	M1
	Attempts to substitute $(x+y)=\pm6$ into $(x+y)^3+10y^2=108x$ to form an equation in y (or x)	dM1
	Solves $216+10y^2=108(6-y)$ and finds the negative root	ddM1
	Awrt 13900 metres	A1
		(4)
		(9 marks)

(a) Allow use of y' for $\dfrac{dy}{dx}$. This is a proof. Look carefully for evidence of candidates working backwards.

B1: Differentiates $10y^2$ to $20y\dfrac{dy}{dx}$

M1: Attempts to differentiate $(x+y)^3$ Via the chain rule look for $k(x+y)^2\left(1+\dfrac{dy}{dx}\right)$ See **

Alternatively you will see attempts where $(x+y)^3$ is multiplied out. Terms may not be collected but

look for 3 of $x^3\to...x^2$, $...x^2y\to2xy+x^2\dfrac{dy}{dx}$, $...xy^2\to y^2+2xy\dfrac{dy}{dx}$ and $y^3\to3y^2\dfrac{dy}{dx}$

differentiated to the correct form.

A1: Correct differentiation $3(x+y)^2\left(1+\dfrac{dy}{dx}\right)+20y\dfrac{dy}{dx}=108$

or $3x^2+3\left(2xy+x^2\dfrac{dy}{dx}\right)+3\left(y^2+2xy\dfrac{dy}{dx}\right)+3y^2\dfrac{dy}{dx}+20y\dfrac{dy}{dx}=108$

dM1: Collects the terms in $\dfrac{dy}{dx}$.

Look for either $\dfrac{dy}{dx}$ being factorised out of the relevant terms

or the terms in $\dfrac{dy}{dx}$ being set on one side of the equation and the other terms on the other side

It is dependent upon **BOTH t**he B and M marks.

A1*: Proceeds to the given answer.