Question	Scheme	Marks
8(a)	$V = \pi \int_0^{10} \left(10xe^{-\frac{1}{2}x}\right)^2 \mathrm{d}x =; \; 100\pi \int_0^{10} x^2 \mathrm{e}^{-x} \; \mathrm{d}x$	M1;A1
		(2)
(b)	$\int x^2 \mathrm{e}^{-x} \; \mathrm{d}x = -x^2 \mathrm{e}^{-x} + 2\int x \mathrm{e}^{-x} \; \mathrm{d}x$	M1
	$= -x^2 \mathrm{e}^{-x} + 2\int x \mathrm{e}^{-x} \; \mathrm{d}x = -x^2 \mathrm{e}^{-x} + 2\left\{-x\mathrm{e}^{-x} + \int \mathrm{e}^{-x} \mathrm{d}x\right\}$	dM1
	$-x^2 \mathrm{e}^{-x} - 2x\mathrm{e}^{-x} + 2\int \mathrm{e}^{-x} \mathrm{d}x = -x^2 \mathrm{e}^{-x} - 2x\mathrm{e}^{-x} - 2\mathrm{e}^{-x} \; (+c)$	A1
		(3)
(c)	Total volume $= 2 \times "100\pi" \int_0^{10} x^2 \mathrm{e}^{-x} \mathrm{d}x$	M1
	$\int_0^{10} x^2 \mathrm{e}^{-x} = \left[-x^2 \mathrm{e}^{-x} - 2x\mathrm{e}^{-x} - 2\mathrm{e}^{-x}\right]_0^{10} = \left(-(10)^2 \mathrm{e}^{-10} - 2 \times 10\mathrm{e}^{-10} - 2\mathrm{e}^{-10}\right) - \left(-2\right)$	M1
	$= 2 - 122\mathrm{e}^{-10} \quad (1.9944\ldots)$	A1
	$\text{Density} = \dfrac{5000}{"200\pi" \times "1.9944..."}$	dM1
	awrt 3.99 (g / cm^3)	A1
		(5)
		(10 marks)

Notes

(a)

M1: Forms a correct unsimplified expression for the volume.

A1: Achieves $100\pi \int_0^{10} x^2 \mathrm{e}^{-x} \; \mathrm{d}x$ Condone a missing dx but limits must be present.

(b)

M1: Attempts integration by parts in the right direction to achieve an expression of the form $\ldots x^2 \mathrm{e}^{-x} \pm \ldots \int x \mathrm{e}^{-x} \; \mathrm{d}x$ Condone missing dx

dM1: Dependent on the previous method mark. Attempts integration by parts a second time to achieve an expression of the form $\ldots x^2 \mathrm{e}^{-x} \pm \ldots x\mathrm{e}^{-x} \pm \ldots \int \mathrm{e}^{-x} \; \mathrm{d}x$ Condone missing dx

A1: $-x^2 \mathrm{e}^{-x} - 2x\mathrm{e}^{-x} - 2\mathrm{e}^{-x} \; (+c)$ with or without the constant of integration

For attempts via the DI (tabular) method, look for first two rows of the table to have correct forms for M1, all rows with correct forms and answer extracted for dM1 and A1 for correct answer.

(c)

M1: A correct strategy to find the total volume with their values of *k*.

M1: Substitutes the limits of 10 and 0 into their part (b) and subtracts. Alternatively allow M1 for limits 20 and 0 used (as a mistaken attempt to double).

A1: $2 - 122\mathrm{e}^{-10}$ or awrt 1.99

dM1: Dependent on second M. Attempts to find the density using $\dfrac{5000}{\text{their Volume}}$. The attempt at the volume need not be correct but an attempt at using (b) must have been made. E.g. if they forget *k* or forget to double, allow for the attempt with their volume. Must be with 5000 in numerator, or with correct work to reach correct units later.

A1: awrt 3.99 (g / cm^3) oe. Accept exact simplified answers such as $\dfrac{5000}{200\pi(2 - 122\mathrm{e}^{-10})}$

Question	Scheme	Marks
9	For question 9 many variations on the proof are possible. Below is a general outline with some examples, which cover many cases. If you see an approach you do not know how to score, consult your team leader. **M1:** Will be scored for setting up an algebraic statement in terms of a variable (integer) *k* or any other variable aside *n* that engages with divisibility by 4 in some way and can lead to a contradiction and is scored at the point you can see each of these elements. A formal statement of the assumption is not required at this stage. **A1:** Scored for a correct statement from which it is possible to draw a contradiction. **dM1;** For making a complete argument that leads to a (full) contradiction of the initial statement, though may be allowed if there are minor gaps or omissions. **A1:** Correct and complete work with contradiction drawn and conclusion made. There must have been a statement of assumption at the start for which to draw the contradiction, though it may not be technicality a correct assumption as long as a relevant assumption has been made. E.g. Accept "Assume n^2-2 is divisible be 4 for all *n"*	
9	(Assume that there is an *n* with n^2-2 is divisible by 4 so) $n^2-2=4k$	M1
	then $n^2=4k+2=2(2k+1)$ (so is even)	A1
	Hence n^2 is even so *n* (=2*m*) is even hence n^2 is a multiple of 4 As n^2 is a multiple of 4 then $n^2-2=4m^2-2=2(2m^2-1)$ cannot be a multiple of 4 (as $2m-1$ is odd) so there is a contradiction.	dM1
	So the original assumption has been shown false. Hence "n^2-2 is never divisible by 4" is true for all *n* *	A1*
		(4)
		(4 marks)

Notes

M1: Sets up an algebraic statement in terms of a variable (integer) *k* or any other variable aside *n* that engages with divisibility by 4 in some way and can lead to a contradiction. No need for explicit statement of assumption - accept if just a suitable equation is set up. In this case supposing divisibility by 4 by stating $n^2-2=4k$

A1: Reaches $n^2=2(2k+1)$

dM1: For a complete argument that leads to a contradiction. See scheme. Allow if minor details are omitted as long as the overall argument is clear.

Accept explanations such as "as n^2 is even then *n* is even hence n^2 is a multiple of 4 so n^2-2 cannot be a multiple of 4 (as 4 does not divide 2)"

A1*: Draws the contradiction to their initial assumption and concludes the statement is true for all *n*. There must have been a clear assumption at the start that is contradicted, and all working must have been correct. For the assumption be generous with the technicality as long as a relevant assumption has been made. E.g. Accept "Assume n^2-2 is divisible be 4 for all *n"*

9 Alt 1	(Assume that n^2-2 is divisible by 4 for some *n*,) so $\frac{n^2-2}{4}$ is an integer. Then if *n* is even $n = 2m$ (*m* integer) so $\frac{n^2-2}{4}=\frac{(2m)^2-2}{4}$ (oe with odd)	M1
	$=m^2-\frac{1}{2}$ (which is not an integer)	A1

	Since m^2 is an integer, $m^2-\frac{1}{2}$ is not, hence n cannot be even, but if n is odd then $\frac{n^2-2}{4}=\frac{(2m+1)^2-2}{4}=m^2+m-\frac{1}{4}$, which is again not an integer (since $m^2 + m$ is)	dM1
	Hence there is a contradiction (as n cannot be an integer) Hence "n^2-2 is never divisible by 4" is true for all n *	A1*
		(4)
		(4 marks)

Notes

M1: Sets up an algebraic statement in terms of a variable (integer) m or any other variable aside n that engages with divisibility by 4 in some way and can lead to a contradiction. No need for explicit statement of assumption - accept if just a suitable equation is set up. In this Alt, consider case use of $n = 2m$ or $n = 2m + 1$ in $\frac{n^2-2}{4}$ is sufficient

A1: Reaches $m^2-\frac{1}{2}$ for n even or $m^2+m-\frac{1}{4}$ for n odd.

dM1: For a complete argument that leads to a contradiction in both cases. See scheme. Allow if minor details are omitted as long as the overall argument is clear.

A1*: Draws the contradiction to their initial assumption and concludes the statement is true for all n. There must have been a clear assumption at the start that is contradicted, and all working must have been correct. For the assumption be generous with the technicality as long as a relevant assumption has been made. E.g. Accept "Assume n^2-2 is divisible be 4 for all n"

9 Alt 2	(Assume that n^2-2 is divisible by 4) $\Rightarrow n^2-2=4k$	M1
	$\Rightarrow n^2=4k+2\Rightarrow n=2\sqrt{k+\frac{1}{2}}$ or $n=\sqrt{2}\sqrt{2k+1}$	A1
	So for some integer m $\sqrt{k+\frac{1}{2}}=\frac{m}{2}\Rightarrow 2k+1=\frac{m^2}{2}$ but m^2 is odd if m is odd so $\frac{m^2}{2}$ not an integer, or m^2 is a multiple of 4 if m even, so odd=even or $2k + 1$ is odd, so does not have a factor 2 to combine with the $\sqrt{2}$ outside, hence n must be irrational	dM1
	Hence we have a contradiction. So "n^2-2 is never divisible by 4" is true for all n *	A1*
		(4)
		(4 marks)

Notes

M1: Sets up an algebraic statement in terms of a variable (integer) k or any other variable aside n that engages with divisibility by 4 in some way and can lead to a contradiction. No need for explicit statement of assumption - accept if just a suitable equation is set up. In this case supposing divisibility by 4 by stating $n^2-2=4k$

A1: Reaches $n=2\sqrt{k+\frac{1}{2}}$ or $n=\sqrt{2}\sqrt{2k+1}$

dM1: For a complete argument that leads to a contradiction. See scheme. Allow if minor details are omitted as long as the overall argument is clear. Must be a valid attempt to show that $2\sqrt{k+\frac{1}{2}}$ / $\sqrt{2}\sqrt{2k+1}$ is not an integer, and this method is a hard route.

A1*: Draws the contradiction to their initial assumption and concludes the statement is true for all *n*. There must have been a clear assumption at the start that is contradicted, and all working must have been correct. For the assumption be generous with the technicality as long as a relevant assumption has been made. E.g. Accept "Assume n^2-2 is divisible be 4 for all *n"*

9 Alt 3	(Assume that n^2-2 is divisible by 4) then for *n* even we have (for some integer *m*) $n^2-2=4m^2-2$ **or** for *n* odd $n^2-2=4\left(m^2+m\right)-1$	M1
	$4m^2-2$ or $4\left(m^2+m\right)-1$	A1
	Since 4 divides n^2-2 and $4m^2$ thus for *n* even, 4 must divide 2, a contradiction, so *n* cannot be even, and also 4 divides $4\left(m^2+m\right)$ so for *n* odd, 4 divides 1, also a contradiction.	dM1
	Hence we have a contradiction for both cases (and as *n* must be either even or odd). so "n^2-2 is never divisible by 4" is true for all *n* *	A1*
		(4)
		(4 marks)

Notes

M1: Sets up an algebraic statement in terms of a variable (integer) *m* or any other variable aside *n* that engages with divisibility by 4 in some way and can lead to a contradiction. No need for explicit statement of assumption - accept if just a suitable equation is set up. In this case supposing using *n* odd or *n* even to form an expression for n^2-2 of the form $4\times\text{integer}\pm\text{non-mulitple of 4}$

A1: Reaches $4m^2-2$ or $4\left(m^2+m\right)-1$

dM1: For a complete argument that leads to a contradiction. See scheme. Allow if minor details are omitted as long as the overall argument is clear. Both cases must be considered with a reason for the contradiction given (not just stated not divisible by 4).

A1*: Draws the contradiction to their initial assumption and concludes the statement is true for all *n*. There must have been a clear assumption at the start that is contradicted, and all working must have been correct. For the assumption be generous with the technicality as long as a relevant assumption has been made. E.g. Accept "Assume n^2-2 is divisible be 4 for all *n"*