8. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

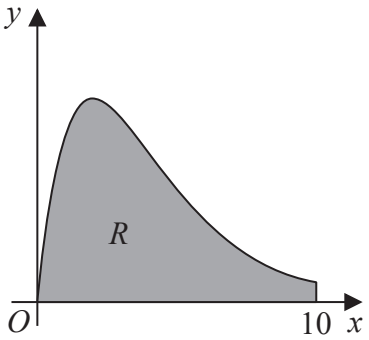

Figure 2

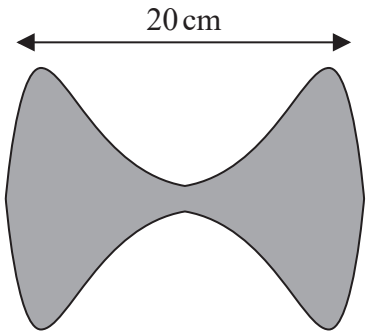

Figure 3

Figure 2 shows the curve with equation

$$y = 10x\mathrm{e}^{-\frac{1}{2}x} \qquad 0 \leqslant x \leqslant 10$$

The finite region R, shown shaded in Figure 2, is bounded by the curve, the x-axis and the line with equation $x = 10$

The region R is rotated through 2π radians about the x-axis to form a solid of revolution.

(a) Show that the volume, V, of this solid is given by

$$V = k\int_0^{10} x^2\mathrm{e}^{-x}\,\mathrm{d}x$$

where k is a constant to be found.

(2)

(b) Find $\int x^2\mathrm{e}^{-x}\,\mathrm{d}x$

(3)

Figure 3 represents an exercise weight formed by joining two of these solids together.

The exercise weight has mass 5 kg and is 20 cm long.

Given that

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

and using your answers to part (a) and part (b),

(c) find the density of this exercise weight. Give your answer in grams per cm^3 to 3 significant figures.

(5)

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Question 8 continued

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Question 8 continued

(Total for Question 8 is 10 marks)

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9. Use proof by contradiction to show that, when n is an integer,

$$n^2 - 2$$

is **never** divisible by 4

(4)

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