Question	Scheme	Marks
7(a)	$\frac{dx}{dt} = \cos t + 6\cos t \sin t \qquad \frac{dy}{dt} = 3\cos t - 2\sin t$	B1B1
	$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{3\cos t - 2\sin t}{\cos t + 6\cos t \sin t} = \frac{3\cos\pi - 2\sin\pi}{\cos\pi + 6\cos\pi\sin\pi} = 3$ *	M1A1*
		(4)
(b)	When $t = \pi$, $x = -3$, $y = -2$	B1
	$y - "-2" = 3(x - "-3")$	M1
	$y = 3x + 7$	A1
		(3)
(c)	$y = 3x + 7 \Rightarrow 3\sin t + 2\cos t = 3\left(\sin t - 3\cos^2 t\right) + 7$ or $y = 3\left(x + 3\cos^2 t\right) + 2\cos t \Rightarrow 3x + 7 = 3x + 9\cos^2 t + 2\cos t$	M1
	$\Rightarrow 9\cos^2 t + 2\cos t - 7 = 0$ *	A1*
		(2)
(d)	$\cos t = \frac{7}{9}$	B1
	$y = 3 \times \frac{\sqrt{32}}{9} + 2 \times \frac{7}{9} = \frac{4\sqrt{2}}{3} + \frac{14}{9}$	M1A1
		(3)
		(12 marks)

Notes

(a)

B1: $\left(\frac{dx}{dt} =\right) \cos t + 6\cos t \sin t$ or $\cos t + 3\sin 2t$

B1: $\left(\frac{dy}{dt} =\right) 3\cos t - 2\sin t$

M1: Attempts $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$ using their $\frac{dx}{dt}$ and their $\frac{dy}{dt}$ and substitutes $t = \pi$. (May substitute π before dividing.)

A1*: Achieves $\frac{dy}{dx} = 3$ with full working shown and no errors.

(b)

B1: $x = -3$, $y = -2$ which may be seen within their working

M1: Attempts to find the equation of the tangent with gradient 3. If they use $y = mx + c$ they must proceed as far as $c = ...$

A1: $y = 3x + 7$

(c)

M1: A full attempt to solve simultaneously the given parametric equations with their equation of the tangent

A1*: Achieves $9\cos^2 t + 2\cos t - 7 = 0$ with no errors

(d)

B1: $\cos t = \frac{7}{9}$ seen or implied. Allow if seen in (c).

M1: Attempts to find the *y* coordinate Must attempt to evaluate trig terms. If no substitution/working shown, then score for awrt 3.44 following a correct value for cos *t*

A1: $\frac{4\sqrt{2}}{3} + \frac{14}{9}$ or exact equivalent. Withhold if additional answers are given.

Question	Scheme	Marks
8(a)	$V = \pi \int_0^{10} \left(10xe^{-\frac{1}{2}x}\right)^2 \mathrm{d}x =; \; 100\pi \int_0^{10} x^2 \mathrm{e}^{-x} \; \mathrm{d}x$	M1;A1
		(2)
(b)	$\int x^2 \mathrm{e}^{-x} \; \mathrm{d}x = -x^2 \mathrm{e}^{-x} + 2\int x \mathrm{e}^{-x} \; \mathrm{d}x$	M1
	$= -x^2 \mathrm{e}^{-x} + 2\int x \mathrm{e}^{-x} \; \mathrm{d}x = -x^2 \mathrm{e}^{-x} + 2\left\{-x\mathrm{e}^{-x} + \int \mathrm{e}^{-x} \mathrm{d}x\right\}$	dM1
	$-x^2 \mathrm{e}^{-x} - 2x\mathrm{e}^{-x} + 2\int \mathrm{e}^{-x} \mathrm{d}x = -x^2 \mathrm{e}^{-x} - 2x\mathrm{e}^{-x} - 2\mathrm{e}^{-x} \; (+c)$	A1
		(3)
(c)	Total volume $= 2 \times "100\pi" \int_0^{10} x^2 \mathrm{e}^{-x} \mathrm{d}x$	M1
	$\int_0^{10} x^2 \mathrm{e}^{-x} = \left[-x^2 \mathrm{e}^{-x} - 2x\mathrm{e}^{-x} - 2\mathrm{e}^{-x}\right]_0^{10} = \left(-(10)^2 \mathrm{e}^{-10} - 2 \times 10\mathrm{e}^{-10} - 2\mathrm{e}^{-10}\right) - \left(-2\right)$	M1
	$= 2 - 122\mathrm{e}^{-10} \quad (1.9944\ldots)$	A1
	$\text{Density} = \dfrac{5000}{"200\pi" \times "1.9944..."}$	dM1
	awrt 3.99 (g / cm^3)	A1
		(5)
		(10 marks)

Notes

(a)

M1: Forms a correct unsimplified expression for the volume.

A1: Achieves $100\pi \int_0^{10} x^2 \mathrm{e}^{-x} \; \mathrm{d}x$ Condone a missing dx but limits must be present.

(b)

M1: Attempts integration by parts in the right direction to achieve an expression of the form $\ldots x^2 \mathrm{e}^{-x} \pm \ldots \int x \mathrm{e}^{-x} \; \mathrm{d}x$ Condone missing dx

dM1: Dependent on the previous method mark. Attempts integration by parts a second time to achieve an expression of the form $\ldots x^2 \mathrm{e}^{-x} \pm \ldots x\mathrm{e}^{-x} \pm \ldots \int \mathrm{e}^{-x} \; \mathrm{d}x$ Condone missing dx

A1: $-x^2 \mathrm{e}^{-x} - 2x\mathrm{e}^{-x} - 2\mathrm{e}^{-x} \; (+c)$ with or without the constant of integration

For attempts via the DI (tabular) method, look for first two rows of the table to have correct forms for M1, all rows with correct forms and answer extracted for dM1 and A1 for correct answer.

(c)

M1: A correct strategy to find the total volume with their values of *k*.

M1: Substitutes the limits of 10 and 0 into their part (b) and subtracts. Alternatively allow M1 for limits 20 and 0 used (as a mistaken attempt to double).

A1: $2 - 122\mathrm{e}^{-10}$ or awrt 1.99

dM1: Dependent on second M. Attempts to find the density using $\dfrac{5000}{\text{their Volume}}$. The attempt at the volume need not be correct but an attempt at using (b) must have been made. E.g. if they forget *k* or forget to double, allow for the attempt with their volume. Must be with 5000 in numerator, or with correct work to reach correct units later.

A1: awrt 3.99 (g / cm^3) oe. Accept exact simplified answers such as $\dfrac{5000}{200\pi(2 - 122\mathrm{e}^{-10})}$