Question	Scheme	Marks
4(a)	$16x^3 - 9kx^2y + 8y^3 = 875$	
	$(8)y^3 \to (8\times)3y^2\dfrac{dy}{dx}$	B1
	$-9kx^2y \to ...kxy \pm ... - 9kx^2\dfrac{dy}{dx}$	M1
	$48x^2 - 18kxy - 9kx^2\dfrac{dy}{dx} + 24y^2\dfrac{dy}{dx} = 0 \Rightarrow \dfrac{dy}{dx}(24y^2 - 9kx^2) = 18kxy - 48x^2 \Rightarrow \dfrac{dy}{dx} = ...$	M1
	$\dfrac{dy}{dx} = \dfrac{6kxy - 16x^2}{8y^2 - 3kx^2}$ *	A1*
		(4)
(b)	$\dfrac{dy}{dx} = 0, x = \dfrac{5}{2} \Rightarrow \dfrac{6k\left(\frac{5}{2}\right)y - 16\left(\frac{5}{2}\right)^2}{8y^2 - 3k\left(\frac{5}{2}\right)^2} = 0$ or $x = \dfrac{5}{2} \Rightarrow 16\left(\dfrac{5}{2}\right)^3 - 9k\left(\dfrac{5}{2}\right)^2 y + 8y^3 = 875$	M1
	$15ky - 100 = 0$ or $250 - \dfrac{225}{4}ky + 8y^3 = 875$	A1
	E.g. $16\left(\dfrac{5}{2}\right)^3 - 9k\left(\dfrac{5}{2}\right)^2\left(\dfrac{20}{3k}\right) + 8\left(\dfrac{20}{3k}\right)^3 = 875 \Rightarrow k^3 = ... \left(= \dfrac{64}{27}\right) \Rightarrow k = ...$	M1
	$k = \dfrac{4}{3}$	A1
		(4)
		(8 marks)

Notes

(a)

B1: For $y^3 \to 3y^2 \frac{dy}{dx}$. Allow if seen in aside working without the 8.

M1: Correct attempt at implicit differentiation on the $-9kx^2y$. Look for $-9kx^2y \to ...kxy \pm ... - 9kx^2 \frac{dy}{dx}$

M1: Collects both of their $\frac{dy}{dx}$ terms together, collects non $\frac{dy}{dx}$ terms the other side of the equation, factorises and divides to achieve $\frac{dy}{dx} = ...$ Must have two $\frac{dy}{dx}$ terms, one from the attempt at differentiating $-9kx^2y$ and one from the attempt at differentiating y^3, but condone if an extra $\frac{dy}{dx} = ...$ term has been included.

A1*: Achieves $\frac{dy}{dx} = \frac{6kxy - 16x^2}{8y^2 - 3kx^2}$ with no errors

(b)

M1: Uses the information to produce one equation in *k* and *y*, e.g. sets the $\frac{dy}{dx}$ equal to 0 and substitutes $x = \frac{5}{2}$, or substitutes $x = \frac{5}{2}$ into the given equation. Allow one slip substituting.

A1: A correct equation without fraction and with simplified coefficients, so $15ky - 100 = 0$ oe or $250 - \frac{225}{4}ky + 8y^3 = 875$ oe

M1: For a complete method to find *k* so solves the equations simultaneously to achieve a value for *k*. May find *y* first e.g substitutes their $k = \frac{20}{3y}$ into the original equation, solves to find *y* and substitutes this back into $k = \frac{20}{3y}$ to find *k* via $250 - 375 + 8y^3 = 875 \Rightarrow y = 5 \Rightarrow k = \frac{20}{3 \times 5} = ...$

A1 $k = \frac{4}{3}$

Alt:

If they do not substitute $x = \frac{5}{2}$ initially then score

M1: Uses numerator of $\frac{dy}{dx}$ equal to 0 to find *y* in terms of *x* and *k* and substitute into original equation (allowing one slip)

A1: Correct equation:

$6kxy - 16x^2 = 0 \Rightarrow y = \frac{8x^2}{3kx} \Rightarrow 16x^3 - 9kx^2\left(\frac{8x^2}{3kx}\right) + 8\left(\frac{8x^2}{3kx}\right)^3 = 875$ oe

M1: Substitutes $x = \frac{5}{2}$ and solves to find *k*

A1: $k = \frac{4}{3}$

Question	Scheme	Marks
5(a)	$1=2\sin u \Rightarrow p=\sin^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{6}$	B1
	$x=2\sin u \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}u}=2\cos u$ oe	M1
	$\int \frac{3x+2}{\left(4-x^2\right)^{\frac{3}{2}}}\,\mathrm{d}x=\int \frac{6\sin u+2}{\left(4-4\sin^2 u\right)^{\frac{3}{2}}}\,2\cos u\,\mathrm{d}u=\int \frac{6\sin u+2}{\left(4\cos^2 u\right)^{\frac{3}{2}}}\,2\cos u\,\mathrm{d}u$	M1
	$=\int \frac{12\sin u}{8\cos^2 u}+\frac{2}{4\cos^2 u}\,\mathrm{d}u=\int_0^{\frac{\pi}{6}}\left(\frac{3}{2}\sec u\tan u+\frac{1}{2}\sec^2 u\right)\mathrm{d}u$ *	A1*
		(4)
(b)	$\int \left(\frac{3}{2}\sec u\tan u+\frac{1}{2}\sec^2 u\right)\mathrm{d}u=\frac{3}{2}\sec u+\frac{1}{2}\tan u$	M1A1
	$\left[\frac{3}{2}\sec u+\frac{1}{2}\tan u\right]_0^{"\frac{\pi}{6}"}=\left(\frac{3}{2}\sec\left("\frac{\pi}{6}"\right)+\frac{1}{2}\tan\left("\frac{\pi}{6}"\right)\right)-\left(\frac{3}{2}\sec 0+\frac{1}{2}\tan 0\right)=...$	M1
	$=\sqrt{3}+\frac{\sqrt{3}}{6}-\frac{3}{2}=\frac{7\sqrt{3}}{6}-\frac{3}{2}\ \left(=\frac{7\sqrt{3}-9}{6}\right)$	A1
		(4)
		(8 marks)

Notes

(a)

B1: $p=\frac{\pi}{6}$ Allow if seen anywhere, even in (b). p = 30 is B0.

M1: $x=2\sin u \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}u}=\pm...\cos u$ or any rearrangement of this equation.

M1: **Full** substitution from an integral in terms of x to an integral in terms of u and **uses** the identity $\sin^2 u+\cos^2 u=1$ in the denominator. Do not be concerned with the limits for this mark.

A1*: Achieves given answer include du (with their p) with no errors and at least one intermediate step with the fractional power simplified. Condone missing du in intermediate lines.

(b)

M1: $\int \left(\frac{3}{2}\sec u\tan u+\frac{1}{2}\sec^2 u\right)\mathrm{d}u=...\sec u+...\tan u$

A1: $\frac{3}{2}\sec u+\frac{1}{2}\tan u$ ignore any constant c

M1: Depends on having one term of the correct form, attempts to substitute in their p ($\neq 1$) and 0, subtracting either way round. The substitution must be seen or clearly implied, e.g. by correct values for each term in an intermediate step before the answer (allowing missing 0's).

A1: $\frac{7\sqrt{3}}{6}-\frac{3}{2}$ or exact equivalent eg $\frac{7\sqrt{3}-9}{6}$ Allow if p = 30° was used.