Question	Scheme	Marks
8(a)	Attempts $\pm\left(\begin{pmatrix}6\\0\\7\end{pmatrix}-\begin{pmatrix}6\\6\\2\end{pmatrix}\right)=\pm\begin{pmatrix}0\\-6\\5\end{pmatrix}$	**M1**
	$\mathbf{r}=\begin{pmatrix}6\\6\\2\end{pmatrix}+\lambda\begin{pmatrix}0\\-6\\5\end{pmatrix}$ or $\mathbf{r}=\begin{pmatrix}6\\0\\7\end{pmatrix}+\lambda\begin{pmatrix}0\\6\\-5\end{pmatrix}$ oe	**A1**
		(2)
(b)	Meet if $\begin{pmatrix}6\\6\\2\end{pmatrix}+\lambda\begin{pmatrix}0\\-6\\5\end{pmatrix}=\begin{pmatrix}3\\1\\4\end{pmatrix}+\mu\begin{pmatrix}1\\5\\9\end{pmatrix}$ so $\begin{cases}6=3+\mu\\6-6\lambda=1+5\mu\\2+5\lambda=4+9\mu\end{cases}$	**M1**
	From first equation $\mu=3$ Alt: solves equations 2 and 3 to give either $\mu=\frac{13}{79}$ or $\lambda=\frac{55}{79}$	**A1**
	Then need $\lambda=\frac{5-5\times3}{6}=-\frac{5}{3}$ from 2nd equation, $\lambda=\frac{2+9\times3}{5}=\frac{29}{5}$ from 3rd equation Alt: checks their μ in the first equation	**M1**
	Values of λ do not agree and hence lines do not meet.	**A1**
		(4)
(c)	$\overrightarrow{OC}=\begin{pmatrix}3\\1\\4\end{pmatrix}-\left(\begin{pmatrix}1\\5\\9\end{pmatrix}\left(=\begin{pmatrix}2\\-4\\-5\end{pmatrix}\right)\right)$	**M1**
	$\overrightarrow{AC}=\overrightarrow{OC}-\overrightarrow{OA}=\begin{pmatrix}2\\-4\\-5\end{pmatrix}-\begin{pmatrix}6\\6\\2\end{pmatrix}=\begin{pmatrix}-4\\-10\\-7\end{pmatrix}$ Accept $\pm$. **Depends on first mark.**	**dM1**
	$\frac{\pm\overrightarrow{AC}\bullet(\mathbf{i}+5\mathbf{j}+9\mathbf{k})}{\left\lvert\overrightarrow{AC}\right\rvert\lvert\mathbf{i}+5\mathbf{j}+9\mathbf{k}\rvert}=\pm\frac{-4\times1+-10\times5+-7\times9}{\sqrt{16+100+49}\sqrt{1+25+81}}$ **Depends on both previous marks.**	**ddM1A1**
	$\Rightarrow\theta=\arccos\left\lvert\frac{-117}{\sqrt{165}\sqrt{107}}\right\rvert=28.3°$ or e.g. $\Rightarrow\theta=90°-\arcsin\left\lvert\frac{-117}{\sqrt{165}\sqrt{107}}\right\rvert=28.3°$	**A1**
		(5)
		(11 marks)

Notes:

(a)

M1: Attempts the difference between the given two vectors. Implied by 2 out of three correct coordinates if no method is shown. Can be either way round.

A1: Any correct equation for the line in the form given and **must have** "**r** = …". Two examples are shown in the scheme, but they may use any point on the line as the starting point, and any multiple of the direction vector.

Allow in **i**, **j**, **k** form but not e.g. $\mathbf{r} = \begin{pmatrix} 6\mathbf{i} \\ 6\mathbf{j} \\ 2\mathbf{k} \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ -6\mathbf{j} \\ 5\mathbf{k} \end{pmatrix}$ but condone missing brackets on the position vector so condone e.g. $\mathbf{r} = \begin{matrix} 6 \\ 6 \\ 2 \end{matrix} + \lambda \begin{pmatrix} 0 \\ -6 \\ 5 \end{pmatrix}$ but not on the direction so $\mathbf{r} = \begin{pmatrix} 6 \\ 6 \\ 2 \end{pmatrix} + \lambda \begin{matrix} 0 \\ -6 \\ 5 \end{matrix}$ is A0

(b)

M1: Equates their line and the given line and extracts at least one equation in λ **and** μ

A1: **"**Correct" value for one parameter. This will depend on which equation(s) they solve – see main scheme **but must follow M1**.

M1: Substitutes into both other equations to compare results.

A1: Correct values for λ found with conclusion that the lines do not meet.

Alt: If equation 1 is not first used to find μ then the 1st A mark can be awarded for any correct value for λ or μ from the equations they solve, and the M mark for checking their solutions for consistency in the remaining equation.

For the final A, all work must have been correct with a suitable conclusion made.

When checking, allow e.g. $2 - \frac{25}{3} \neq 4 + 27$ **so they do not meet. I.e. the calculations may not be fully carried out as long as the result is obvious. However if either side is subsequently evaluated incorrectly, withhold the final mark.**

FYI: equations (1) and (2) give $\lambda = -\frac{5}{3}$ or $\mu = 3$; equations (1) and (3) give $\lambda = \frac{29}{5}$ or $\mu = 3$ and equations (2) and (3) give $\lambda = \frac{55}{79}$ or $\mu = \frac{13}{79}$ (use of the negative direction vectors reverse the signs of λs)

If $\mathbf{r} = \begin{pmatrix} 6 \\ 0 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 6 \\ -5 \end{pmatrix}$ is used equations (1) and (2) give $\lambda = \frac{8}{3}$ or $\mu = 3$; equations (1) and (3) give $\lambda = -\frac{24}{5}$ or $\mu = 3$ and equations (2) and (3) give $\lambda = \frac{24}{79}$ or $\mu = \frac{13}{79}$ (use of the negative direction vectors reverse the signs of λs)

Note that the maximum marks in (b) if their line in (a) is incorrect is 1010

There will be many different approaches to part (b).
The general structure for marking is as follows:
M1: Equates lines and obtains at least one equation in λ and μ
A1: Correct value for one of the parameters
M1: Attempts the checking process to show the lines do not meet
A1: All correct with a conclusion

(c)

M1: Substitutes $\mu = -1$ into l_2 to find the coordinates for C or the vector $\overrightarrow{OC}$

dM1: Uses their $\overrightarrow{OC}$ to find $\pm\overrightarrow{AC}$. Look for an attempt at the difference of vectors with at least two correct coordinates.

ddM1: Attempts $\dfrac{\pm\overrightarrow{AC}\bullet(\mathbf{i}+5\mathbf{j}+9\mathbf{k})}{\left|\overrightarrow{AC}\right|\left|\mathbf{i}+5\mathbf{j}+9\mathbf{k}\right|}$ with their $\overrightarrow{AC}$

This requires an attempt at the scalar product of their $\overrightarrow{AC}$ with $\mathbf{i}+5\mathbf{j}+9\mathbf{k}$ in the numerator with at least 2 "components" correct if no method is shown, and correct attempts at the product of the magnitudes of $\overrightarrow{AC}$ and the direction of $\mathbf{i}+5\mathbf{j}+9\mathbf{k}$ in the denominator.

A1: A correct expression simplified or unsimplified e.g. $\pm\left(\dfrac{-4\times1+-10\times5+-7\times9}{\sqrt{16+100+49}\sqrt{1+25+81}}\right)$, $\pm\dfrac{117}{\sqrt{17655}}$

A1: Correct answer. Accept awrt 28.3

Question	Scheme	Marks
9(a)	$\frac{\mathrm{d}}{\mathrm{d}y}\left(1+2\ln y\right)^{-2} = \alpha\left(1+2\ln y\right)^{-3} \times \frac{\beta}{y} = \frac{K}{y\left(1+2\ln y\right)^{3}}$	M1
	$= \frac{-4}{y\left(1+2\ln y\right)^{3}}$ oe	A1
		(2)
(b)	$3\text{cosec}(2x)\frac{\mathrm{d}y}{\mathrm{d}x} = y\left(1+2\ln y\right)^{3}$ $\Rightarrow \int ...\mathrm{d}y = \int k\sin(2x)\,\mathrm{d}x$ **or** $\int \frac{k}{y\left(1+2\ln y\right)^{3}}\mathrm{d}y = \int ...\mathrm{d}x$ **and** $\Rightarrow \int ...\mathrm{d}y = \int k\sin(2x)\,\mathrm{d}x \Rightarrow ... = A\cos 2x$ oe **or** $\Rightarrow \int \frac{k}{y\left(1+2\ln y\right)^{3}}\mathrm{d}y = \int ...\mathrm{d}x \Rightarrow \frac{A}{\left(1+2\ln y\right)^{2}} = ...$	M1
	$\Rightarrow \int \frac{A}{y\left(1+2\ln y\right)^{3}}\mathrm{d}y = \int B\sin(2x)\,\mathrm{d}x \Rightarrow \frac{C}{\left(1+2\ln y\right)^{2}} = D\cos 2x$ oe	M1
	One side integrated **correctly** $\int \frac{3}{y\left(1+2\ln y\right)^{3}}\mathrm{d}y = -\frac{3}{4\left(1+2\ln y\right)^{2}}, \int \frac{1}{y\left(1+2\ln y\right)^{3}}\mathrm{d}y = -\frac{1}{4\left(1+2\ln y\right)^{2}}$ or $\int \frac{1}{3}\sin(2x)\,\mathrm{d}x = -\frac{1}{6}\cos(2x)$ oe, $\int \sin(2x)\,\mathrm{d}x = -\frac{1}{2}\cos(2x)$ oe	A1
	$-\frac{3}{4\left(1+2\ln y\right)^{2}} = -\frac{1}{2}\cos(2x)\left(+c\right)$ or $-\frac{1}{4\left(1+2\ln y\right)^{2}} = -\frac{1}{6}\cos(2x)\left(+c\right)$	A1
		(4)
(c)	$y=1, x=\frac{\pi}{6} \Rightarrow -\frac{3}{4\left(1+2\ln 1\right)} = -\frac{1}{2}\cos\left(\frac{\pi}{3}\right)+c \Rightarrow c = ...$	M1
	$-\frac{3}{4\left(1+2\ln y\right)^{2}} = -\frac{1}{2}\cos(2x)-\frac{1}{2}$ or $-\frac{1}{4\left(1+2\ln y\right)^{2}} = -\frac{1}{6}\cos(2x)-\frac{1}{6}$	A1
	$-\frac{3}{4\left(1+2\ln y\right)^{2}} = -\frac{1}{2}\cos(2x)-\frac{1}{2} \Rightarrow \frac{3}{4\left(1+2\ln y\right)^{2}} = \frac{1}{2}+\frac{1}{2}\cos\left(2x\right) = \cos^{2}x$	M1
	$\Rightarrow \left(1+2\ln y\right)^{2} = \frac{3}{4}\sec^{2}x \Rightarrow \ln y = \frac{1}{2}\left(\frac{\sqrt{3}}{2}\sec x - 1\right)$ **Depends on both previous method marks.**	ddM1
	$\Rightarrow y = \mathrm{e}^{\frac{\sqrt{3}}{4}\sec x - \frac{1}{2}}$	A1
		(5)

(11 marks)

Notes:

(a)

M1: Differentiates to achieve the form $\dfrac{K}{y(1+2\ln y)^3}$.

You can ignore what they call it so just look for the expression.

A1: Correct derivative, accept any equivalent form, need not be simplified.

You can ignore what they call it so just look for the expression.

(b)

M1: Attempts to separate the variables with one side correct ignoring coefficients and with at least one side integrated to the correct form. The separation may have $\dfrac{k}{\text{cosec}\,2x}$ rather than $k\sin 2x$.

Do not be concerned about the presence or absence of the "dx" and "dy" or even if they are wrongly position – just look for the expressions appearing on either side.

M1: Attempt to separate the variables with both sides correct ignoring coefficients and with both sides integrated to the correct form. The separation may have $\dfrac{k}{\text{cosec}\,2x}$ rather than $k\sin 2x$.

Do not be concerned about the presence or absence of the "dx" and "dy" or even if they are wrongly position – just look for the expressions appearing on either side.

A1: One side integrated **correctly**.

For the LHS this will be either $-\dfrac{1}{4(1+2\ln y)^2}$ or $-\dfrac{3}{4(1+2\ln y)^2}$

For the RHS this will be either $-\dfrac{1}{2}\cos(2x)$ or $-\dfrac{1}{6}\cos(2x)$ but note that if sin 2x is changed to 2sin x cos x first, the integration may appear as e.g. $\dfrac{1}{3}\sin^2 x$ or $-\dfrac{1}{3}\cos^2 x$ or $\sin^2 x$ or $-\cos^2 x$

A1: A **fully correct** general solution, need not be simplified and a constant of integration is not required.

Note that M1M0A1A0 is possible as a mark profile.

(c)

Note that if a candidate rearranges to an explicit form in part (b) the work can be credited in part (c) but there is no credit for work for part (c) in part (b) unless part (c) is attempted.

M1: Substitutes the boundary values into their equation to find their constant which does not to be evaluated for this mark.

This may be done later in the solution, but they must have a constant in the equation.

A1: Correct equation in any form with their constant evaluated. **Must follow A1 in (b).**

M1: Applies cos $2x = \pm 2\cos^2 x \pm 1$ at some point in their solution.

May see attempts to rearrange to $y = \ldots$ first **and note that if candidates integrate the sin 2x in part (b) to …cos^2x then they can score this mark by implication if part (c) is attempted.**

ddM1: Rearranges the equation, **including square rooting**, to reach ln$y$ = …

Depends on both previous method marks.

A1: Correct answer achieved.

You may see candidates rearrange first in (b) to achieve e.g.