Question	Scheme	Marks
5(a)	$\dfrac{\mathrm{d}x}{\mathrm{d}t}=\dfrac{1}{2}(9-4t)^{-\frac{1}{2}}\times -4=\left(\dfrac{-2}{\sqrt{9-4t}}\right)$	**M1**
	$A=\displaystyle\int_{(x=0)}^{(x=3)} y\dfrac{\mathrm{d}x}{\mathrm{d}t}(\mathrm{d}t)=\int_{\left(t=\frac{9}{4}\right)}^{(t=0)}\dfrac{t^3}{\sqrt{9+4t}}\times \text{"}\dfrac{-2}{\sqrt{9-4t}}\text{"}(\mathrm{d}t)$	**M1**
	$=-2\displaystyle\int_{\left(\frac{9}{4}\right)}^{(0)}\dfrac{t^3}{\sqrt{81-16t^2}}(\mathrm{d}t)$ or e.g. $-2\displaystyle\int_{\left(\frac{9}{4}\right)}^{(0)}\dfrac{t^3}{\sqrt{(9-4t)(9+4t)}}(\mathrm{d}t)$ **Depends on both previous M marks.**	**ddM1**
	$=2\displaystyle\int_0^{\frac{9}{4}}\dfrac{t^3}{\sqrt{81-16t^2}}\mathrm{d}t$	**A1**
		(4)
(b)	$\dfrac{\mathrm{d}u}{\mathrm{d}t}=-32t$ or $\mathrm{d}u=-32t\,\mathrm{d}t$ oe or $u=81-16t^2\Rightarrow t^2=\dfrac{1}{16}(81-u)\Rightarrow t=\dfrac{1}{4}(81-u)^{\frac{1}{2}}\Rightarrow\dfrac{\mathrm{d}t}{\mathrm{d}u}=-\dfrac{1}{8}(81-u)^{-\frac{1}{2}}$	**B1**
	$A=2\displaystyle\int_0^{\frac{9}{4}}\dfrac{t^2}{\sqrt{81-16t^2}}t\mathrm{d}t=2\int_{(81)}^{(0)}\dfrac{(81-u)}{16\sqrt{u}}\dfrac{\mathrm{d}u}{-32}$ or $A=2\displaystyle\int_0^{\frac{9}{4}}\dfrac{t^2}{\sqrt{81-16t^2}}t\mathrm{d}t=2\int_{(81)}^{(0)}\dfrac{\left(\frac{1}{4}(81-u)^{\frac{1}{2}}\right)^3}{\sqrt{u}}\times-\dfrac{1}{8}(81-u)^{-\frac{1}{2}}\mathrm{d}u$	**M1**
	$=k\displaystyle\int_{(81)}^{(0)}81u^{-\frac{1}{2}}-u^{\frac{1}{2}}\mathrm{d}u$	**A1ft**
	$=K\left[\dfrac{81u^{\frac{1}{2}}}{\frac{1}{2}}-\dfrac{u^{\frac{3}{2}}}{\frac{3}{2}}\right]_{(81)}^{(0)}$	**M1**
	$=-\dfrac{1}{256}\left(0-\left(162\sqrt{81}-\dfrac{2}{3}\sqrt{81}^3\right)\right)=\ldots$ **Depends on all previous M marks.**	**ddM1**
	$=\dfrac{243}{64}$	**A1**
		(6)
		(10 marks)
Notes:		

(a)

M1: Attempts $\frac{\mathrm{d}x}{\mathrm{d}t}$ achieving the form $k\left(9-4t\right)^{-\frac{1}{2}}$ oe

M1: Applies the correct formula for the area, $A=\int y\frac{\mathrm{d}x}{\mathrm{d}t}\mathrm{d}t$, with y and their $\frac{\mathrm{d}x}{\mathrm{d}t}$

(no need for limits for this mark and no need for "dt").

ddM1: Combines terms to a single fraction with one square root.

(no need for limits for this mark and no need for "dt").

Depends on both previous M marks.

A1: Applies difference of squares, or expands, and deals with limits correctly to achieve correct result from correct working.

The "dt" must have made at least one appearance before the final line.

(b)

B1: Correct statement of $\frac{\mathrm{d}u}{\mathrm{d}t}$ or $\frac{\mathrm{d}t}{\mathrm{d}u}$ or any correct equation connecting du with dt

M1: Complete substitution made into the integrand given in part (a) with their K or the letter K or a made up K e.g. $K = 1$ to obtain an integral in terms of u only.

Don't be concerned with the limits at this stage.

Condone the omission of the "du".

A1ft: For any multiple of $\int\left(81u^{-\frac{1}{2}}-u^{\frac{1}{2}}\right)\mathrm{d}u.$ Limits are not needed.

Condone the omission of the "du".

M1: Integrates an expression of the form $ku^{-\frac{1}{2}}+lu^{\frac{1}{2}}\rightarrow Au^{\frac{1}{2}}+Bu^{\frac{3}{2}}$

ddM1: Applies correct limits to their integral either way round. Correct limits are 0 and 81 if substituting into their integral in terms of u or 0 and $\frac{9}{4}$ if reverting to t.

Depends on all previous M marks.

A1: Correct answer. Correct work leading to $-\frac{243}{64}$ scores A0 but allow "hence area is $\frac{243}{64}$"

Note that the question says "or otherwise" so some alternatives are:

Uses the substitution $u=\sqrt{81-16t^2}$:

B1: Correct derivative statement $\frac{\mathrm{d}u}{\mathrm{d}t}=\frac{1}{2}\left(81-16t^2\right)^{-\frac{1}{2}}\times-32t$ oe such as $2u\,\mathrm{d}u=-32t\,\mathrm{d}t$

M1A1ft: Full substitution leading to $2\int\frac{81-u^2}{16u}\frac{2u}{-32}\mathrm{d}u=-\frac{1}{128}\int81-u^2\,\mathrm{d}u$

ddM1: Integrates to $Ku-Lu^3$ **M1:** Applies correct limits 0 and 81 (or returns to t)

Depends on all previous M marks.

A1: Correct.

Integration by parts:

B1: For $\int\frac{t}{\sqrt{81-16t^2}}\mathrm{d}t=-\frac{1}{16}\sqrt{81-16t^2}$ seen or implied in working.

M1A1ft: Correct application of parts in the right direction

$$2\int t^2 \times \frac{t}{\sqrt{81-16t^2}}\mathrm{d}t = 2t^2 \times -\frac{1}{16}\sqrt{81-16t^2} - 2\int 2t \times -\frac{1}{16}\sqrt{81-16t^2}\ \mathrm{d}t$$

M1: Completes the integration process $= -\frac{t^2}{8}\sqrt{81-16t^2} + \frac{1}{4} \times -\frac{\sqrt{81-16t^2}^3}{48}$

ddM1: Applies limits of 0 and $\frac{9}{4}$. **Depends on all previous M marks.**

A1: Correct answer.

Question	Scheme	Marks
6	Assume the sequence is geometric	B1
	So $\left(r=\right)\dfrac{1+2k}{k}=\dfrac{3+3k}{1+2k}$	M1
	$\Rightarrow (1+2k)^2=k(3+3k)\Rightarrow k^2+k+1=0$	A1
	But $k^2+k+1=\left(k+\dfrac{1}{2}\right)^2-\dfrac{1}{4}+1\geqslant\dfrac{3}{4}>0$ (since $\left(k+\dfrac{1}{2}\right)^2\geqslant 0$ for all (real) k)	dM1
	This is a contradiction and hence the original assumption is not true. The sequence is not geometric.	A1
		(5)
		(5 marks)

Notes:

(a)

B1: States an appropriate assumption to set up the contradiction.

M1: Uses the assumption to set up an equation in k only.

Allow equivalent work e.g. $kr=1+2k,\ kr^2=3+3k\Rightarrow 3+3k=k\left(\dfrac{1+2k}{k}\right)^2$

Allow use of $\neq$ for = e.g. $\sqrt{\dfrac{3+3k}{k}}\neq\dfrac{1+2k}{k}$

This may be implied by e.g. $\dfrac{1+2k}{k}$ is not the same as $\dfrac{3+3k}{1+2k}$.

A1: Reaches a correct quadratic equation in k, need not be all on one side, but terms in k and k^2 should be collected. Allow use of $\neq$ for = e.g. $k^2+k+1\neq 0$

dM1: Completes the square, considers the discriminant or other valid means used to reach a point where a contradiction can be deduced. E.g. as scheme, or $b^2-4ac=1^2-4(1)(1)=-3<0$ may be used. Accept use of calculator to give roots $k=\dfrac{-1\pm i\sqrt{3}}{2}$ so k is not real, which contradicts k being a member of the real sequence.

Depends on the previous M.

A1: Correct work leading to a contradiction with deduction of a contradiction made and conclusion given. **This mark is available even if B0 is given at the start. So 01111 is possible.**

If they are using the discriminant or calculator route then there is no need to mention "real" as long as they conclude that e.g. the geometric sequence is not possible. This can score both the dM1 and A1.