Question	Scheme	Marks
4(a)	Area of one face $= \frac{1}{2}x \times x \times \sin 60° = \frac{1}{2}x^2\frac{\sqrt{3}}{2}$ oe	M1
	So surface area of icosahedron is $20 \times \frac{\sqrt{3}}{4}x^2 = 5\sqrt{3}x^2$ *	A1*
		(2)
(b)	$\frac{dA}{dx} = 10\sqrt{3}x$ and $\frac{dV}{dx} = \frac{15}{12}\left(3+\sqrt{5}\right)x^2$	M1
	$\frac{dV}{dA} = \frac{dV}{dx} \times \frac{dx}{dA} = \frac{dV}{dx} \div \frac{dA}{dx} = \ldots$ (oe)	M1
	$= \frac{15\left(3+\sqrt{5}\right)x^2}{12 \times 10\sqrt{3}x} = \frac{\left(3+\sqrt{5}\right)x}{8\sqrt{3}}$ *	A1*
		(3)
(c)	$\frac{dA}{dt} = 0.025$	B1
	$\frac{dV}{dt} = \frac{dV}{dA} \times \frac{dA}{dt} = \frac{\left(3+\sqrt{5}\right)x}{8\sqrt{3}} \times 0.025 = \ldots$ When $x = 2$, $\frac{dV}{dt} = \frac{2\left(3+\sqrt{5}\right)}{8\sqrt{3}} \times 0.025 = \ldots$	M1
	awrt 0.019 (cm^3s^{-1})	A1
		(3)
		(8 marks)

Notes:

(a)

M1: Any correct method to establish a surd form for the area of one face. Must evaluate trigonometric terms.

Alternatives include e.g. $\frac{1}{2} \times x \times \sqrt{x^2 - \left(\frac{1}{2}x\right)^2} = \frac{1}{2}x\sqrt{\frac{3}{4}x^2}$

A1*: Correct result achieved after showing side area of one face.

Examples:

Area of one face $= \frac{1}{2}x \times x \times \sin 60° = \frac{1}{4}x^2\sqrt{3} \Rightarrow SA = 20 \times \frac{\sqrt{3}}{4}x^2 = 5\sqrt{3}x^2$ scores M1 A1

$SA = 20 \times \left(\frac{1}{2} \times x \times x \sin 60\right) = 5\sqrt{3}x^2$ scores M1 A0 for lack of working

(b)

M1: Attempts derivatives of both A and V wrt x. E.g. $\left(\frac{dA}{dx} =\right)\alpha x$ and $\left(\frac{dV}{dx} =\right)\beta x^2$.

Note that $\frac{5}{12}\left(3+\sqrt{5}\right)x^3$ is sometimes expanded first as e.g. $\frac{5}{4}x^3 + \frac{5\sqrt{5}}{12}x^3$ and allow this mark if either term is differentiated to βx^2

M1: Applies the correct chain rule with their derivatives. (Condone poor notation – see below)

A1*: Applies the chain rule to achieve the given result, with suitable intermediate step seen with $\frac{\mathrm{d}V}{\mathrm{d}A}=...$ appearing somewhere in their solution.

Condone poor notation e.g. allow $\frac{\mathrm{d}A}{\mathrm{d}t}=10\sqrt{3}x$ and $\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{15}{12}\left(3+\sqrt{5}\right)x^2$, $\frac{\mathrm{d}V}{\mathrm{d}A}=\frac{\mathrm{d}V}{\mathrm{d}t}\times\frac{\mathrm{d}t}{\mathrm{d}A}$ etc. for full marks.

Alt:

M1: Eliminates x from the two equations $V=\frac{5}{12}\left(3+\sqrt{5}\right)\left(\frac{A}{5\sqrt{3}}\right)^{\frac{3}{2}}$

M1: Attempts to differentiate $\frac{\mathrm{d}V}{\mathrm{d}A}=\frac{5}{12}\left(3+\sqrt{5}\right)\times\frac{3}{2}\left(\frac{A}{5\sqrt{3}}\right)^{\frac{1}{2}}\times\frac{1}{5\sqrt{3}}$ – look for correct treatment of the $5\sqrt{3}$ as well as correct power.

A1: Achieves correct result with no errors.

(c)

B1: Interprets the rate of change correctly to state or clearly imply by working that $\frac{\mathrm{d}A}{\mathrm{d}t}=0.025$

M1: Applies the chain rule appropriately **to find a value for** $\frac{\mathrm{d}V}{\mathrm{d}t}$ using $x=2$

A1: awrt 0.019 condone lack of units.

Question	Scheme	Marks
5(a)	$\dfrac{\mathrm{d}x}{\mathrm{d}t}=\dfrac{1}{2}(9-4t)^{-\frac{1}{2}}\times -4=\left(\dfrac{-2}{\sqrt{9-4t}}\right)$	**M1**
	$A=\displaystyle\int_{(x=0)}^{(x=3)} y\dfrac{\mathrm{d}x}{\mathrm{d}t}(\mathrm{d}t)=\int_{\left(t=\frac{9}{4}\right)}^{(t=0)}\dfrac{t^3}{\sqrt{9+4t}}\times \text{"}\dfrac{-2}{\sqrt{9-4t}}\text{"}(\mathrm{d}t)$	**M1**
	$=-2\displaystyle\int_{\left(\frac{9}{4}\right)}^{(0)}\dfrac{t^3}{\sqrt{81-16t^2}}(\mathrm{d}t)$ or e.g. $-2\displaystyle\int_{\left(\frac{9}{4}\right)}^{(0)}\dfrac{t^3}{\sqrt{(9-4t)(9+4t)}}(\mathrm{d}t)$ **Depends on both previous M marks.**	**ddM1**
	$=2\displaystyle\int_0^{\frac{9}{4}}\dfrac{t^3}{\sqrt{81-16t^2}}\mathrm{d}t$	**A1**
		(4)
(b)	$\dfrac{\mathrm{d}u}{\mathrm{d}t}=-32t$ or $\mathrm{d}u=-32t\,\mathrm{d}t$ oe or $u=81-16t^2\Rightarrow t^2=\dfrac{1}{16}(81-u)\Rightarrow t=\dfrac{1}{4}(81-u)^{\frac{1}{2}}\Rightarrow\dfrac{\mathrm{d}t}{\mathrm{d}u}=-\dfrac{1}{8}(81-u)^{-\frac{1}{2}}$	**B1**
	$A=2\displaystyle\int_0^{\frac{9}{4}}\dfrac{t^2}{\sqrt{81-16t^2}}t\mathrm{d}t=2\int_{(81)}^{(0)}\dfrac{(81-u)}{16\sqrt{u}}\dfrac{\mathrm{d}u}{-32}$ or $A=2\displaystyle\int_0^{\frac{9}{4}}\dfrac{t^2}{\sqrt{81-16t^2}}t\mathrm{d}t=2\int_{(81)}^{(0)}\dfrac{\left(\frac{1}{4}(81-u)^{\frac{1}{2}}\right)^3}{\sqrt{u}}\times-\dfrac{1}{8}(81-u)^{-\frac{1}{2}}\mathrm{d}u$	**M1**
	$=k\displaystyle\int_{(81)}^{(0)}81u^{-\frac{1}{2}}-u^{\frac{1}{2}}\mathrm{d}u$	**A1ft**
	$=K\left[\dfrac{81u^{\frac{1}{2}}}{\frac{1}{2}}-\dfrac{u^{\frac{3}{2}}}{\frac{3}{2}}\right]_{(81)}^{(0)}$	**M1**
	$=-\dfrac{1}{256}\left(0-\left(162\sqrt{81}-\dfrac{2}{3}\sqrt{81}^3\right)\right)=\ldots$ **Depends on all previous M marks.**	**ddM1**
	$=\dfrac{243}{64}$	**A1**
		(6)
		(10 marks)
Notes:		