(a) Way 4	$y=\dfrac{6}{7+\cos 2t}\Rightarrow \cos 2t=\dfrac{6}{y}-7\Rightarrow 1-2\sin^2 t=\dfrac{6}{y}-7$	**M1**
	$\sin t=\dfrac{x-3}{2}\Rightarrow \dfrac{6}{y}-7=1-2\left(\dfrac{x-3}{2}\right)^2$	**M1A1**
	$\Rightarrow \dfrac{12}{y}=-x^2+6x+7\Rightarrow y=\dfrac{12}{-x^2+6x+7}=\dfrac{12}{(7-x)(1+x)}$*	**M1A1***
	$\left(t=-\dfrac{\pi}{2}\Rightarrow x=1, t=\dfrac{\pi}{2}\Rightarrow x=5 \text{ so}\right)$ $p = 1$ and $q = 5$	**B1**
(b)	$\dfrac{12}{(7-x)(1+x)}=\dfrac{A}{7-x}+\dfrac{B}{1+x}\Rightarrow 12=A(x+1)+B(7-x)\Rightarrow A=...,B=...$	**M1**
	So $(y=)\dfrac{3}{2(x+1)}-\dfrac{3}{2(x-7)}$ oe	**A1A1**
		(3)
		(9 marks)

Notes:

(a)

M1: Applies $\cos 2t=\pm 1\pm 2\sin^2 t$ to get y in terms of sin t. This may be implied if they obtain cos $2t$ in terms of x with no explicit sight of sin t.

M1: Rearranges equation for x and substitutes into equation for y.

A1: Correct equation in terms of x and y only.

M1: Simplifies and factorises denominator (usual rules). May have the $\frac{1}{2}$ left in the denominator, e.g.

$$y=\frac{6}{8-\frac{1}{2}(x-3)^2}=\frac{6}{8-\frac{1}{2}x^2+3x-\frac{9}{2}}=\frac{6}{-\frac{1}{2}x^2+3x+\frac{7}{2}}=\frac{6}{-\left(\frac{1}{2}x+\frac{1}{2}\right)(x-7)}$$

A1*: Correct result reached with no errors seen.

B1: Correct values for p and q or correct inequality.

Can score anywhere in their response but do not allow $1 \leqslant t \leqslant 5$

(a) Way 2

M1: Applies $\cos 2t=\pm 1\pm 2\sin^2 t$ to get y in terms of sin t

M1: Uses the difference of 2 squares in the denominator

A1: Correct equation with the square completed

M1: Substitutes for sin t in terms of x

A1*: Correct result reached with no errors seen.

B1: Correct values for p and q or correct inequality. **Can score anywhere in their response.**

(a) Way 3

M1: Substitutes for x in terms of t into the given expression for y

M1: Expands denominator.

A1: Correct expression.

M1: Applies $\cos 2t=\pm 1\pm 2\sin^2 t$ to get $\sin^2 t$ in terms of cos $2t$

A1*: Correct result reached with no errors seen and minimal conclusion.

B1: Correct values for p and q or correct inequality. **Can score anywhere in their response.**

(a) Way 4 (similar to Way 1)

M1: Rearranges to get cos $2t$ in terms of y and uses $\cos 2t=\pm 1\pm 2\sin^2 t$

M1: Rearranges equation for x and substitutes to obtain and equation in x and y.

A1: Correct equation in terms of x and y only.

M1: Simplifies and factorises denominator (usual rules).

A1*: Correct result reached with no errors seen.

B1: Correct values for p and q or correct inequality. **Can score anywhere in their response.**

(b)

M1: Correct overall method to apply partial fractions to **the given fraction or equivalent correct fraction** and find at least one constant.

If no method is shown, one correct value can imply the method.

Score M0 for $\dfrac{12}{(7-x)(1+x)}=\dfrac{A}{7-x}+\dfrac{B}{1+x}\Rightarrow 12=A(7-x)+B(x+1)\Rightarrow A=...,B=...$

A1: At least one correct fraction in the required form e.g.

$\dfrac{3}{2(1+x)}$, $-\dfrac{3}{2(x-7)}$ oe e.g. $\dfrac{\frac{3}{2}}{1+x}$, $-\dfrac{\frac{6}{4}}{x-7}$ but not e.g. $\dfrac{3}{2x+2}$, $\dfrac{-3}{2x-14}$

unless a correct form is seen beforehand.

Or a correct pair of values e.g. $a=\dfrac{3}{2},b=1$ or $c=\dfrac{3}{2},d=1$ or $a=-\dfrac{3}{2},b=-7$ or $c=-\dfrac{3}{2},d=-7$

A1: Correct expression for y in the given form $\dfrac{3}{2(x+1)}-\dfrac{3}{2(x-7)}$ (there is no need for the "$y=$")

Accept as in scheme or any equivalents in the correct form, e.g. with 1.5 or equivalent as the numerators.

Or all values correct e.g. $\left(a=\dfrac{3}{2},b=1 \text{ and } c=-\dfrac{3}{2},d=-7\right)$ or $\left(a=-\dfrac{3}{2},b=-7 \text{ and } c=\dfrac{3}{2},d=1\right)$

Note that in (b) some candidates may change the form of y first:

$$\frac{12}{(7-x)(1+x)}=\frac{-12}{(x-7)(1+x)}$$

In such cases the marks can be applied as in the scheme provided the expression is correct.

Question	Scheme	Marks
4(a)	Area of one face $= \frac{1}{2}x \times x \times \sin 60° = \frac{1}{2}x^2\frac{\sqrt{3}}{2}$ oe	M1
	So surface area of icosahedron is $20 \times \frac{\sqrt{3}}{4}x^2 = 5\sqrt{3}x^2$ *	A1*
		(2)
(b)	$\frac{dA}{dx} = 10\sqrt{3}x$ and $\frac{dV}{dx} = \frac{15}{12}\left(3+\sqrt{5}\right)x^2$	M1
	$\frac{dV}{dA} = \frac{dV}{dx} \times \frac{dx}{dA} = \frac{dV}{dx} \div \frac{dA}{dx} = \ldots$ (oe)	M1
	$= \frac{15\left(3+\sqrt{5}\right)x^2}{12 \times 10\sqrt{3}x} = \frac{\left(3+\sqrt{5}\right)x}{8\sqrt{3}}$ *	A1*
		(3)
(c)	$\frac{dA}{dt} = 0.025$	B1
	$\frac{dV}{dt} = \frac{dV}{dA} \times \frac{dA}{dt} = \frac{\left(3+\sqrt{5}\right)x}{8\sqrt{3}} \times 0.025 = \ldots$ When $x = 2$, $\frac{dV}{dt} = \frac{2\left(3+\sqrt{5}\right)}{8\sqrt{3}} \times 0.025 = \ldots$	M1
	awrt 0.019 (cm^3s^{-1})	A1
		(3)
		(8 marks)

Notes:

(a)

M1: Any correct method to establish a surd form for the area of one face. Must evaluate trigonometric terms.

Alternatives include e.g. $\frac{1}{2} \times x \times \sqrt{x^2 - \left(\frac{1}{2}x\right)^2} = \frac{1}{2}x\sqrt{\frac{3}{4}x^2}$

A1*: Correct result achieved after showing side area of one face.

Examples:

Area of one face $= \frac{1}{2}x \times x \times \sin 60° = \frac{1}{4}x^2\sqrt{3} \Rightarrow SA = 20 \times \frac{\sqrt{3}}{4}x^2 = 5\sqrt{3}x^2$ scores M1 A1

$SA = 20 \times \left(\frac{1}{2} \times x \times x \sin 60\right) = 5\sqrt{3}x^2$ scores M1 A0 for lack of working

(b)

M1: Attempts derivatives of both A and V wrt x. E.g. $\left(\frac{dA}{dx} =\right)\alpha x$ and $\left(\frac{dV}{dx} =\right)\beta x^2$.

Note that $\frac{5}{12}\left(3+\sqrt{5}\right)x^3$ is sometimes expanded first as e.g. $\frac{5}{4}x^3 + \frac{5\sqrt{5}}{12}x^3$ and allow this mark if either term is differentiated to βx^2

M1: Applies the correct chain rule with their derivatives. (Condone poor notation – see below)