Question	Scheme	Marks
2(a)	$\left(1+4x^3\right)^{\frac{1}{3}} = 1+\frac{1}{3}\left(...x^{\cdots}\right)+...$	M1
	$\left(1+4x^3\right)^{\frac{1}{3}} = ...+\frac{\frac{1}{3}\left(-\frac{2}{3}\right)}{2}\left(4x^3\right)^2+...$	M1
	$=1+\frac{4}{3}x^3+...$ or $=...-\frac{16}{9}x^6$	A1
	$\left(1+4x^3\right)^{\frac{1}{3}} = 1+\frac{4}{3}x^3-\frac{16}{9}x^6+...$	A1
		(4)
(b)	$1+4x^3 = 1+4\times\left(\frac{1}{3}\right)^3 = \frac{31}{...}$ **or** $1+\frac{4}{3}\left(\frac{1}{3}\right)^3-\frac{16}{9}\left(\frac{1}{3}\right)^6$	M1
	$\sqrt[3]{31} \approx 3\times\left(1+\frac{4}{3}\left(\frac{1}{3}\right)^3-\frac{16}{9}\left(\frac{1}{3}\right)^6\right)$	dM1
	$=\frac{6869}{2187}$	A1
		(3)
		(7 marks)

Notes:

(a)

M1: For $1\pm\frac{1}{3}\left(4x^{\cdots}\right)$. Allow if the 4 and/or the power of x is incorrect/missing for this mark.

M1: For the correct structure for the third term. Look for the correct binomial coefficient combined with the correct power of $4x^3$ e.g. $\frac{\frac{1}{3}\left(\frac{1}{3}-1\right)}{2}\left(4x^3\right)^2$ but condone missing brackets e.g. $\frac{\frac{1}{3}\left(\frac{1}{3}-1\right)}{2}4x^{3^2}$

A1: For $1+\frac{4}{3}x^3+...$ **or** for $...-\frac{16}{9}x^6$.

Allow mixed fractions e.g. $1\frac{1}{3}$ for $\frac{4}{3}$ but do not allow '+–' for '–'.

A1: For $1+\frac{4}{3}x^3-\frac{16}{9}x^6+...$. **Ignore any extra terms.**

(b)

M1: Attempts $1+4\times\left(\frac{1}{3}\right)^3$ and reaches $\frac{31}{k}$ (this will usually be seen as e.g. $\sqrt[3]{\frac{31}{27}}$) **or** substitutes $x=\frac{1}{3}$ into their expansion from part (a).

dM1: Attempts $3\times$ (their expansion from part (a) with $x=\frac{1}{3}$ substituted)

Allow if they have included more terms in their expansion for both M's. You need to be convinced that they have attempted this calculation and not just found the cube root of 31 on their calculator e.g. 3.1414…

A1: Cao. Correct fraction.

Question	Scheme	Marks
3(a)	$x = 3 + 2\sin t \qquad y = \dfrac{6}{7+\cos 2t} \qquad -\dfrac{\pi}{2} \leqslant t \leqslant \dfrac{\pi}{2}$	
	$y = \dfrac{6}{7+1-2\sin^2 t}$	**M1**
	$\sin t = \dfrac{x-3}{2} \Rightarrow y = \dfrac{6}{8-2\left(\dfrac{x-3}{2}\right)^2}$	**M1A1**
	$\Rightarrow y = \dfrac{12}{16-(x-3)^2} = \dfrac{12}{(4-x+3)(4+x-3)} = \dfrac{12}{(7-x)(1+x)}$*	**M1A1***
	$\left(t = -\dfrac{\pi}{2} \Rightarrow x = 1, t = \dfrac{\pi}{2} \Rightarrow x = 5 \text{ so}\right)$ $p = 1$ and $q = 5$	**B1**
		(6)
(a) Way 2	$y = \dfrac{6}{7+1-2\sin^2 t}$	**M1**
	$y = \dfrac{12}{16-4\sin^2 t} = \dfrac{12}{(4+2\sin t)(4-2\sin t)}$	**M1A1**
	$= \dfrac{12}{(4+x-3)(4-(x-3))} = \dfrac{12}{(7-x)(1+x)}$*	**M1A1***
	$\left(t = -\dfrac{\pi}{2} \Rightarrow x = 1, t = \dfrac{\pi}{2} \Rightarrow x = 5 \text{ so}\right)$ $p = 1$ and $q = 5$	**B1**
(a) Way 3	$y = \dfrac{12}{(7-x)(1+x)} = \dfrac{12}{(4-2\sin t)(4+2\sin t)}$	**M1**
	$= \dfrac{12}{16-4\sin^2 t}$	**M1A1**
	$= \dfrac{12}{16-4\left(\dfrac{1-\cos 2t}{2}\right)}$	**M1**
	$= \dfrac{12}{16-2+2\cos 2t} = \dfrac{12}{14+2\cos 2t} = \dfrac{6}{7+\cos 2t} = y$*	**A1***
	$\left(t = -\dfrac{\pi}{2} \Rightarrow x = 1, t = \dfrac{\pi}{2} \Rightarrow x = 5 \text{ so}\right)$ $p = 1$ and $q = 5$	**B1**

(a) Way 4	$y=\dfrac{6}{7+\cos 2t}\Rightarrow \cos 2t=\dfrac{6}{y}-7\Rightarrow 1-2\sin^2 t=\dfrac{6}{y}-7$	**M1**
	$\sin t=\dfrac{x-3}{2}\Rightarrow \dfrac{6}{y}-7=1-2\left(\dfrac{x-3}{2}\right)^2$	**M1A1**
	$\Rightarrow \dfrac{12}{y}=-x^2+6x+7\Rightarrow y=\dfrac{12}{-x^2+6x+7}=\dfrac{12}{(7-x)(1+x)}$*	**M1A1***
	$\left(t=-\dfrac{\pi}{2}\Rightarrow x=1, t=\dfrac{\pi}{2}\Rightarrow x=5 \text{ so}\right)$ $p = 1$ and $q = 5$	**B1**
(b)	$\dfrac{12}{(7-x)(1+x)}=\dfrac{A}{7-x}+\dfrac{B}{1+x}\Rightarrow 12=A(x+1)+B(7-x)\Rightarrow A=...,B=...$	**M1**
	So $(y=)\dfrac{3}{2(x+1)}-\dfrac{3}{2(x-7)}$ oe	**A1A1**
		(3)
		(9 marks)

Notes:

(a)

M1: Applies $\cos 2t=\pm 1\pm 2\sin^2 t$ to get y in terms of sin t. This may be implied if they obtain cos $2t$ in terms of x with no explicit sight of sin t.

M1: Rearranges equation for x and substitutes into equation for y.

A1: Correct equation in terms of x and y only.

M1: Simplifies and factorises denominator (usual rules). May have the $\frac{1}{2}$ left in the denominator, e.g.

$$y=\frac{6}{8-\frac{1}{2}(x-3)^2}=\frac{6}{8-\frac{1}{2}x^2+3x-\frac{9}{2}}=\frac{6}{-\frac{1}{2}x^2+3x+\frac{7}{2}}=\frac{6}{-\left(\frac{1}{2}x+\frac{1}{2}\right)(x-7)}$$

A1*: Correct result reached with no errors seen.

B1: Correct values for p and q or correct inequality.

Can score anywhere in their response but do not allow $1 \leqslant t \leqslant 5$

(a) Way 2

M1: Applies $\cos 2t=\pm 1\pm 2\sin^2 t$ to get y in terms of sin t

M1: Uses the difference of 2 squares in the denominator

A1: Correct equation with the square completed

M1: Substitutes for sin t in terms of x

A1*: Correct result reached with no errors seen.

B1: Correct values for p and q or correct inequality. **Can score anywhere in their response.**

(a) Way 3

M1: Substitutes for x in terms of t into the given expression for y

M1: Expands denominator.

A1: Correct expression.

M1: Applies $\cos 2t=\pm 1\pm 2\sin^2 t$ to get $\sin^2 t$ in terms of cos $2t$

A1*: Correct result reached with no errors seen and minimal conclusion.

B1: Correct values for p and q or correct inequality. **Can score anywhere in their response.**

(a) Way 4 (similar to Way 1)

M1: Rearranges to get cos $2t$ in terms of y and uses $\cos 2t=\pm 1\pm 2\sin^2 t$