| Question | Scheme | Marks |
|---|---|---|
| 1 | E.g $xy^2 \to (...+)2xy\dfrac{dy}{dx}$ or $x^2y \to (...+)x^2\dfrac{dy}{dx}$ but see notes. | **M1** |
| | $xy^2 \to y^2 + kxy\dfrac{dy}{dx}$ or $x^2y \to kxy + x^2\dfrac{dy}{dx}$ | **dM1** |
| | $y^2 + 2xy\dfrac{dy}{dx} = 2xy + x^2\dfrac{dy}{dx}$ oe | **A1** |
| | $\left(x^2 - 2xy\right)\dfrac{dy}{dx} = y^2 - 2xy \Rightarrow \dfrac{dy}{dx} = \dfrac{y^2 - 2xy}{x^2 - 2xy} \Rightarrow \left.\dfrac{dy}{dx}\right\|_P = \dfrac{3^2 - 12}{2^2 - 12} = ...$ or<br>$3^2 + 2\times2\times3\left.\dfrac{dy}{dx}\right\|_P = 2\times2\times3 + 2^2\left.\dfrac{dy}{dx}\right\|_P \Rightarrow \left.\dfrac{dy}{dx}\right\|_P = \dfrac{9-12}{4-12} = ...$ | **M1** |
| | Tangent is $y - 3 = "\dfrac{3}{8}"(x-2)$ | **dM1** |
| | $\Rightarrow 3x - 8y + 18 = 0$ | **A1** |
| | | **(6)** |
| | | **(6 marks)** |

**Notes:**

**M1:** For an attempt at implicit differentiation in some form. (Allow $y'$ for $\dfrac{dy}{dx}$)

For the $xy^2$ look for a term $kxy\dfrac{dy}{dx}$ but condone $ky\dfrac{dy}{dx}$

For the $x^2y$ look for a term $kx^2\dfrac{dy}{dx}$ but condone $kx\dfrac{dy}{dx}$

**dM1:** For attempting the product rule on either $xy^2$ or $x^2y$.

Award for $xy^2 \to y^2 + kxy\dfrac{dy}{dx}$ **or** $x^2y \to kxy + x^2\dfrac{dy}{dx}$

**A1:** For $y^2 + 2xy\dfrac{dy}{dx} = 2xy + x^2\dfrac{dy}{dx}$ or equivalent.

Ignore any spurious $"\dfrac{dy}{dx} = "$ in front of their equation.

**M1:** Attempts to find $\dfrac{dy}{dx}$ at *P*. Must substitute **both** *x* and *y* values and rearrange (but can be done in either order). Allow this mark as long as there are at least two terms in $\dfrac{dy}{dx}$ in their differentiation not including any spurious $"\dfrac{dy}{dx} = "$ There must be some correct substitution.

**dM1:** Uses their **non-zero** $\dfrac{dy}{dx}$ at (2, 3) to find the equation of the tangent. **Depends on previous M.**

Look for $y - 3 = "m"(x-2)$ or $y = "m"x + c$ where *m* is their attempt at the gradient at *P*, followed by attempt to find *c* using (2, 3) with the 2 and the 3 **positioned correctly.**

**A1:** $3x - 8y + 18 = 0$ or any non-zero integer multiple of this equation but with all terms on one side.

| Question | Scheme | Marks |
|---|---|---|
| 2(a) | $\left(1+4x^3\right)^{\frac{1}{3}} = 1+\frac{1}{3}\left(...x^{\cdots}\right)+...$ | M1 |
| | $\left(1+4x^3\right)^{\frac{1}{3}} = ...+\frac{\frac{1}{3}\left(-\frac{2}{3}\right)}{2}\left(4x^3\right)^2+...$ | M1 |
| | $=1+\frac{4}{3}x^3+...$ or $=...-\frac{16}{9}x^6$ | A1 |
| | $\left(1+4x^3\right)^{\frac{1}{3}} = 1+\frac{4}{3}x^3-\frac{16}{9}x^6+...$ | A1 |
| | | (4) |
| (b) | $1+4x^3 = 1+4\times\left(\frac{1}{3}\right)^3 = \frac{31}{...}$ **or** $1+\frac{4}{3}\left(\frac{1}{3}\right)^3-\frac{16}{9}\left(\frac{1}{3}\right)^6$ | M1 |
| | $\sqrt[3]{31} \approx 3\times\left(1+\frac{4}{3}\left(\frac{1}{3}\right)^3-\frac{16}{9}\left(\frac{1}{3}\right)^6\right)$ | dM1 |
| | $=\frac{6869}{2187}$ | A1 |
| | | (3) |
| | | **(7 marks)** |

**Notes:**

**(a)**

**M1:** For $1\pm\frac{1}{3}\left(4x^{\cdots}\right)$. Allow if the 4 and/or the power of $x$ is incorrect/missing for this mark.

**M1:** For the correct structure for the third term. Look for the correct binomial coefficient combined with the correct power of $4x^3$ e.g. $\frac{\frac{1}{3}\left(\frac{1}{3}-1\right)}{2}\left(4x^3\right)^2$ but condone missing brackets e.g. $\frac{\frac{1}{3}\left(\frac{1}{3}-1\right)}{2}4x^{3^2}$

**A1:** For $1+\frac{4}{3}x^3+...$ **or** for $...-\frac{16}{9}x^6$.

Allow mixed fractions e.g. $1\frac{1}{3}$ for $\frac{4}{3}$ but do not allow '+–' for '–'.

**A1:** For $1+\frac{4}{3}x^3-\frac{16}{9}x^6+...$. **Ignore any extra terms.**

**(b)**

**M1:** Attempts $1+4\times\left(\frac{1}{3}\right)^3$ and reaches $\frac{31}{k}$ (this will usually be seen as e.g. $\sqrt[3]{\frac{31}{27}}$) **or** substitutes $x=\frac{1}{3}$ into their expansion from part (a).

**dM1:** Attempts $3\times$ (their expansion from part (a) with $x=\frac{1}{3}$ substituted)

**Allow if they have included more terms in their expansion for both M's. You need to be convinced that they have attempted this calculation and not just found the cube root of 31 on their calculator e.g. 3.1414…**

**A1:** Cao. Correct fraction.