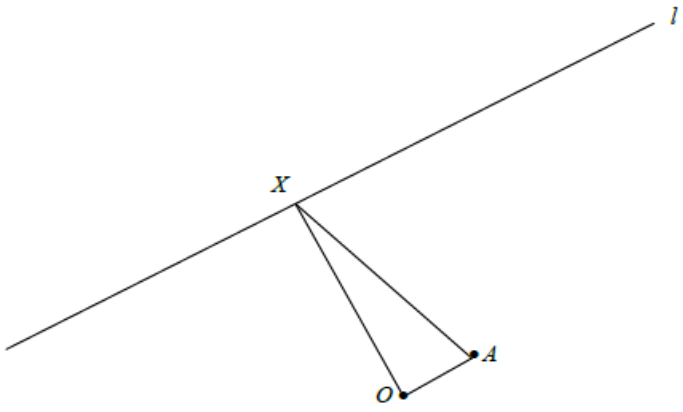
Question Number	Scheme	Marks
7 (a)	Finds $\left\|\begin{pmatrix}4\\4\\2\end{pmatrix}\right\| = \sqrt{4^2+4^2+2^2} = 6$ and attempts $\frac{1}{"6"}\begin{pmatrix}4\\4\\2\end{pmatrix}$	M1
	$\overrightarrow{OA} = \begin{pmatrix}2/3\\2/3\\1/3\end{pmatrix}$ oe	A1
		(2)
(b)	Co-ordinates or position vector of point $X = \begin{pmatrix}1+4\lambda\\-10+4\lambda\\-9+2\lambda\end{pmatrix}$	M1
	$\overrightarrow{OX}.\begin{pmatrix}4\\4\\2\end{pmatrix} = 0 \Rightarrow 4(1+4\lambda)+4(-10+4\lambda)+2(-9+2\lambda) = 0$	dM1
	$36\lambda = 54 \Rightarrow \lambda = 1.5$	ddM1 A1
	$X = (7,-4,-6)$	A1
		(5)
(c)	Finds $OX = \sqrt{7^2+(-4)^2+(-6)^2} = \sqrt{101}$ and $OA = 1$	M1
	Area $OXA = \frac{1}{2}\times 1\times\sqrt{101} = \frac{\sqrt{101}}{2}$	dM1 A1
		(3)
		(10 marks)

Handy diagram



(a)

M1: Correct attempt at the unit vector. Look for an attempt at $\sqrt{4^2+4^2+2^2}$ and use of $\frac{\mathbf{r}}{|\mathbf{r}|}$ where $\mathbf{r} = \begin{pmatrix}4\\4\\2\end{pmatrix}$ or equivalent vector form such as $\mathbf{r} = (4\mathbf{i}+4\mathbf{j}+2\mathbf{k})$. Condone an attempt to find the coordinates of A

A1: $\overrightarrow{OA}=\begin{pmatrix}2/3\\2/3\\1/3\end{pmatrix}$ or equivalent such as $\overrightarrow{OA}=\frac{1}{6}\begin{pmatrix}4\\4\\2\end{pmatrix}$ or $\overrightarrow{OA}=\frac{1}{6}(4\mathbf{i}+4\mathbf{j}+2\mathbf{k})$ but **must be in vector form and not in coordinate form**.

(b)

M1: For an attempt at the co-ordinates or position vector of point $X=$ $(1+4\lambda,-10+4\lambda,-9+2\lambda)$ **or** $\begin{pmatrix}1+4\lambda\\-10+4\lambda\\-9+2\lambda\end{pmatrix}$

dM1: For using $\overrightarrow{OX}.\begin{pmatrix}4\\4\\2\end{pmatrix}=0$ to set up an equation in λ

Alternatively finds $OX^2=(1+4\lambda)^2+(-10+4\lambda)^2+(-9+2\lambda)^2$ and attempts to differentiate and set = 0

May use $OX^2+OA^2=AX^2$ to set up an equation in λ

ddM1: Solves for λ This is dependent upon having scored both previous M's

A1: Correct value for $\lambda=1.5$

A1: Correct coordinates for $X=(7,-4,-6)$. Condone position vector form $7\mathbf{i}-4\mathbf{j}-6\mathbf{k}$ o.e.

(c)

M1: Finds all elements required to calculate the area.

In the main scheme this would be the distance OX or OX^2 AND distance OA or OA^2 (which is 1)

dM1: Correct method of finding the area of OXA = $\frac{1}{2}\times OX\times 1$

A1: Area = $\frac{\sqrt{101}}{2}$ o.e

.....

There are various alternatives for part (c). Amongst others are;

Alt I **via vector product**.

$$\frac{1}{2}\times\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\7&-4&-6\\2/3&2/3&1/3\end{vmatrix}=\frac{1}{2}\times\left|\frac{8}{3}\mathbf{i}-\frac{19}{3}\mathbf{j}+\frac{22}{3}\mathbf{i}\right|=\frac{1}{2}\times\sqrt{\left(\frac{8}{3}\right)^2+\left(-\frac{19}{3}\right)^2+\left(\frac{22}{3}\right)^2}=\frac{\sqrt{101}}{2}$$

M1: For an attempt at $\frac{1}{2}\times\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\7&-4&-6\\2/3&2/3&1/3\end{vmatrix}=\frac{1}{2}\times\left|\frac{8}{3}\mathbf{i}-\frac{19}{3}\mathbf{j}+\frac{22}{3}\mathbf{i}\right|$

dM1: Followed by an attempt at finding the modulus of the resulting vector multiplied by 1/2

Alt II **via scalar products**

M1: Attempts to find all three components required to find the area triangle OXA

E.g. Angle OXA with length of side OX and XA

Alternatively angle OAX with length of side OA and XA

For this to be scored

- appropriate gradient vectors need to be attempted by subtracting
- a correct attempt at using $\mathbf{a}.\mathbf{b}=|\mathbf{a}||\mathbf{b}|\cos\theta$ to find $\cos\theta$ or θ

dM1: Full attempt at area of triangle using $\frac{1}{2}|OX||XA|\sin(OXA)$ or equivalent

Question Number	Scheme	Marks
8 (a)	$\int y^{-\frac{1}{3}}\mathrm{d}y = \int 6x\mathrm{e}^{-2x}\mathrm{d}x$	B1
	$\frac{3}{2}y^{\frac{2}{3}} = -3x\mathrm{e}^{-2x} + \int 3\mathrm{e}^{-2x}\mathrm{d}x$	M1 M1
	$\frac{3}{2}y^{\frac{2}{3}} = -3x\mathrm{e}^{-2x} - \frac{3}{2}\mathrm{e}^{-2x} + c$	dM1 A1
	Substitutes $(0,1) \Rightarrow c = 3$	M1
	$y^2 = \left(-2x\mathrm{e}^{-2x} - \mathrm{e}^{-2x} + 2\right)^3$	A1
		(7)
(b)	As $x \to \infty, \mathrm{e}^{-2x} \to 0$ and $y^2 = (2)^3 \Rightarrow y = 2^{\frac{3}{2}}$	M1 A1
		(2)
		(9 marks)

(a)

B1: Separates the variables either $\int y^{-\frac{1}{3}}\mathrm{d}y = \int 6x\mathrm{e}^{-2x}\mathrm{d}x$ or $\int \frac{1}{6}y^{-\frac{1}{3}}\mathrm{d}y = \int x\mathrm{e}^{-2x}\mathrm{d}x$

Condone with missing integral signs but the dx and dy **must be present** and **in the correct positions**

M1: For integrating the lhs $y^{-\frac{1}{3}} \to y^{\frac{2}{3}}$

M1: For integrating the rhs by parts the right way around. Look for $\int x\mathrm{e}^{-2x}\mathrm{d}x \to \ ..x\mathrm{e}^{-2x} \pm \int ...\mathrm{e}^{-2x}\mathrm{d}x$

dM1: For fully integrating the rhs to obtain $..x\mathrm{e}^{-2x} \pm ...\mathrm{e}^{-2x}$. Depending upon the previous M

A1: Correct integration with or without '+ c'.

Look for $\frac{3}{2}y^{\frac{2}{3}} = -3x\mathrm{e}^{-2x} - \frac{3}{2}\mathrm{e}^{-2x} + c$ or equivalent such as $3y^{\frac{2}{3}} = -6x\mathrm{e}^{-2x} - 3\mathrm{e}^{-2x} + a$

dM1: Must have a "+c " now. Substitutes $(0,1) \Rightarrow c = ...$

It is dependent upon having a correct attempt to integrate one side so M1 or M2 must have been awarded.

A1: CSO $y^2 = \left(-2x\mathrm{e}^{-2x} - \mathrm{e}^{-2x} + 2\right)^3$

(b)

M1: For $\mathrm{e}^{-2x} \to 0 \ \ y^2 = (\text{"2"})^3$

Follow through on their $\mathrm{g}(x)$ in $y^2 = \mathrm{g}(x)$ but $\mathrm{g}(x)$ must be a function of e^{-kx} with $\mathrm{g}(0) \neq 0$

Implied by a correct decimal answer for their $y^2 = \mathrm{g}(x)$

A1: $y = 2^{\frac{3}{2}}$ o.e such as $y = \sqrt{8}$ cso. ISW after sight of this. Condone $y = \pm 2^{\frac{3}{2}}$ o.e.