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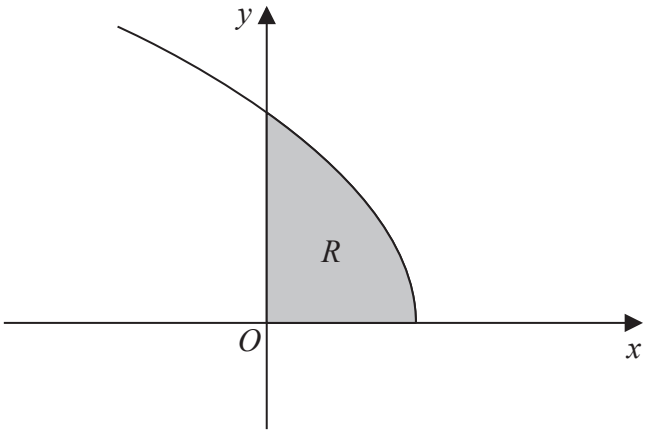

Figure 3

Figure 3 shows a sketch of the curve C with parametric equations

$$x = 2\cos 2t \qquad y = 4\sin t \qquad 0 \leqslant t \leqslant \frac{\pi}{2}$$

The region R, shown shaded in Figure 3, is bounded by the curve, the x-axis and the y-axis.

(a) (i) Show, making your working clear, that the area of $R = \int_0^{\frac{\pi}{4}} 32\sin^2 t \cos t \, \mathrm{d}t$

(ii) Hence find, by algebraic integration, the exact value of the area of R.

(6)

(b) Show that all points on C satisfy $y = \sqrt{ax + b}$, where a and b are constants to be found.

(3)

The curve C has equation $y = \mathrm{f}(x)$ where f is the function

$$\mathrm{f}(x) = \sqrt{ax + b} \qquad -2 \leqslant x \leqslant 2$$

and a and b are the constants found in part (b).

(c) State the range of f.

(1)

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Question 6 continued

Q6

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7. Relative to a fixed origin O, the line l has equation

$$\mathbf{r} = \begin{pmatrix} 1 \\ -10 \\ -9 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 4 \\ 2 \end{pmatrix} \quad \text{where } \lambda \text{ is a scalar parameter}$$

Given that $\overrightarrow{OA}$ is a unit vector parallel to l,

(a) find $\overrightarrow{OA}$

(2)

The point X lies on l.

Given that X is the point on l that is closest to the origin,

(b) find the coordinates of X.

(5)

The points O, X and A form the triangle OXA.

(c) Find the exact area of triangle OXA.

(3)

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