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5. A curve has equation

$$y^2 = y\mathrm{e}^{-2x} - 3x$$

(a) Show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2y\mathrm{e}^{-2x} + 3}{\mathrm{e}^{-2x} - 2y}$$

(4)

The curve crosses the y-axis at the origin and at the point P.

The tangent to the curve at the origin and the tangent to the curve at P meet at the point R.

(b) Find the coordinates of R.

(5)

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Question 5 continued

Q5

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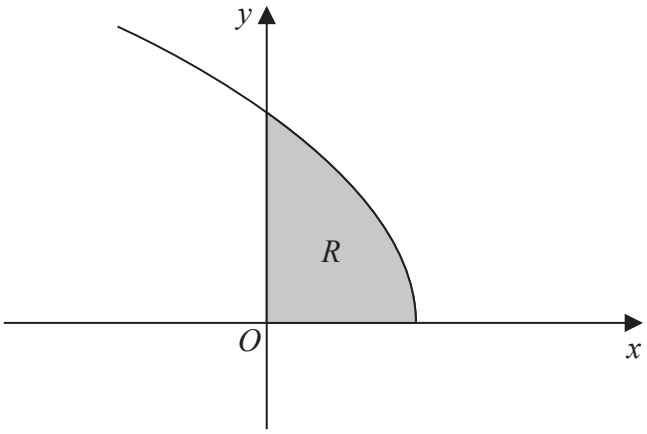


Figure 3

Figure 3 shows a sketch of the curve C with parametric equations

$$x = 2\cos 2t \qquad y = 4\sin t \qquad 0 \leqslant t \leqslant \frac{\pi}{2}$$

The region R, shown shaded in Figure 3, is bounded by the curve, the x-axis and the y-axis.

(a) (i) Show, making your working clear, that the area of $R = \int_0^{\frac{\pi}{4}} 32\sin^2 t \cos t \, \mathrm{d}t$

(ii) Hence find, by algebraic integration, the exact value of the area of R.

(6)

(b) Show that all points on C satisfy $y = \sqrt{ax + b}$, where a and b are constants to be found.

(3)

The curve C has equation $y = \mathrm{f}(x)$ where f is the function

$$\mathrm{f}(x) = \sqrt{ax + b} \qquad -2 \leqslant x \leqslant 2$$

and a and b are the constants found in part (b).

(c) State the range of f.

(1)

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