Question	Scheme	Marks
4.	$\int_1^4 \frac{10}{5x+2x\sqrt{x}}\,\mathrm{d}x$	
	$u=\sqrt{x} \Rightarrow x=u^2 \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}u}=2u$	B1
	$\int \frac{10}{5x+2x\sqrt{x}}\,\mathrm{d}x = \int \frac{10}{5u^2+2u^3}2u\mathrm{d}u$	M1 A1
	$= \int \frac{20}{5u+2u^2}\mathrm{d}u = \int \frac{4}{u}-\frac{8}{5+2u}\mathrm{d}u$	dM1 A1
	$= 4\ln u - 4\ln(5+2u)$	ddM1
	$\text{Limits} = \left[4\ln u - 4\ln(5+2u)\right]_1^2 = 4\ln 2 - 4\ln 9 + 4\ln 7 = \ldots$	M1
	$= 4\ln\left(\frac{14}{9}\right)$	A1 **(8 marks)**

B1: For $\frac{\mathrm{d}x}{\mathrm{d}u}=2u$ or $\frac{\mathrm{d}u}{\mathrm{d}x}=\frac{1}{2}x^{-\frac{1}{2}}$ or equivalent

M1: Attempts to write all terms in the integral in terms of u (inc the dx)

Condone slips for this mark BUT the dx CANNOT just be replaced by du.

Look for either $x \to u^2$ or $x\sqrt{x} \to u^3$ with either $\mathrm{d}x \to \mathrm{f}(u)\mathrm{d}u$ or $\mathrm{d}x \to k \times \mathrm{d}u, k \neq 1$

A1: Correct integrand in terms of just u which may be unsimplified. E.g $\int \frac{10}{5u^2+2u^3}2u\mathrm{d}u$

dM1: Attempts to use PF and writes their integrand in terms of its component fractions.

Look for an integral of the form $\int \frac{P}{Qu+Ru^2}\mathrm{d}u \to \int \frac{p}{u}+\frac{q}{Q+Ru}\mathrm{d}u$

A1: Correct PF $\int \frac{4}{u}-\frac{8}{5+2u}(\mathrm{d}u)$

ddM1: For $\ldots\ln u \pm \ldots\ln(5+2u)$ but follow through on their PF's which must be of a similar form

M1: Uses the limits 1 and 2 within their attempted integral. The integration may be incorrect.

Alternatively substitutes $u=\sqrt{x}$ and uses the limits 1 and 4 within their attempted integral

A1: $4\ln\left(\frac{14}{9}\right)$

Question Number	Scheme	Marks
5 (a)	$2y\frac{dy}{dx} = e^{-2x}\frac{dy}{dx} - 2ye^{-2x} - 3$ $\left(e^{-2x} - 2y\right)\frac{dy}{dx} = 2ye^{-2x} + 3 \Rightarrow \frac{dy}{dx} = \frac{2ye^{-2x}+3}{e^{-2x}-2y}$ *	B1 M1 A1 A1* **(4)**
(b)	Puts $x = 0$ into the equation of the curve $\Rightarrow y = y^2 \Rightarrow y = 1$ Attempts tangent at $(0,0)$ **or** $(0,1)$ $\quad y = 3x$ **or** $y = -5x+1$ Solves $y = 3x$ with $y = -5x+1 \Rightarrow R = \left(\frac{1}{8}, \frac{3}{8}\right)$	B1 M1 A1 dM1 A1 **(5)** **(9 marks)**

(a) Originally scored M1 B1 A1 A1, now B1 M1 A1 A1

B1: Correct differentiation using the chain rule $y^2 \rightarrow 2y\frac{dy}{dx}$.

M1: Attempts to apply the product rule of differentiation on ye^{-2x} to give $e^{-2x}\frac{dy}{dx} \pm ...ye^{-2x}$

A1: Correct differentiation $2y\frac{dy}{dx} = e^{-2x}\frac{dy}{dx} - 2ye^{-2x} - 3$

Allow $2y\,dy = e^{-2x}dy - 2ye^{-2x}dx - 3dx$

A1*: Proceeds to the given answer via an intermediate line equivalent to $\left(e^{-2x} - 2y\right)\frac{dy}{dx} = 2ye^{-2x} + 3$ with correct bracketing.

(b)

B1: Deduces or implies that $x = 0, y = 1$ at P . May state or use coordinates of P (0,1)

M1: Scored for an attempt to find the equation of the tangent at O **or** the equation of the tangent at P

E.g. Substitutes $(0,0)$ into $\frac{dy}{dx} = \frac{2ye^{-2x}+3}{e^{-2x}-2y} = \beta$ and states $y = \beta x$

Alternatively substitutes $(0,"1")$ into $\frac{dy}{dx} = \frac{2ye^{-2x}+3}{e^{-2x}-2y} \rightarrow "k"$ and attempts $y - 1 = "k"x$ or equivalent

A1: Achieves a correct equation for **either** tangent. Look for **either** $y = 3x$ **or** $y = -5x+1$ o.e.

dM1: Correct attempt **at both** tangents with an attempt to solve simultaneously.

For the attempt to solve accept $y = 3x$, $y = -5x+1 \Rightarrow x = ..., y = ...$

A1: Correct coordinates for $R = \left(\frac{1}{8}, \frac{3}{8}\right)$ oe