Question	Scheme	Marks
7 (i)	Chooses a suitable substitution for $\int_1^5 \frac{3x}{\sqrt{2x-1}}\,\mathrm{d}x$	
	E.g. $u=\sqrt{2x-1}$	B1
	$u=\sqrt{2x-1} \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}u}=u$	
	$\int \frac{3x}{\sqrt{2x-1}}\,\mathrm{d}x$	
	$=\int \frac{3}{2}u^2+\frac{3}{2}\,\mathrm{d}u$	M1
	$=\frac{1}{2}u^3+\frac{3}{2}u$	dM1 A1
	$\text{Limits}=\left[\frac{1}{2}u^3+\frac{3}{2}u\right]_1^3=\ldots$	M1
	$\left(\frac{1}{2}\times 3^3+\frac{3}{2}\times 3\right)-\left(\frac{1}{2}\times 1^3+\frac{3}{2}\times 1\right)=16$	A1
		(6)
	E.g. $u=2x-1$	B1
	$u=2x-1 \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}u}=\frac{1}{2}$	
	$\int \frac{3x}{\sqrt{2x-1}}\,\mathrm{d}x$	
	$=\int \frac{3}{4}u^{\frac{1}{2}}+\frac{3}{4}u^{-\frac{1}{2}}\mathrm{d}u$	M1
	$=\frac{1}{2}u^{\frac{3}{2}}+\frac{3}{2}u^{\frac{1}{2}}$	dM1 A1
	$=\left[\frac{1}{2}u^{\frac{3}{2}}+\frac{3}{2}u^{\frac{1}{2}}\right]_1^9=\ldots$	M1
	$\left(\frac{1}{2}\times 9^{\frac{3}{2}}+\frac{3}{2}\times 9^{\frac{1}{2}}\right)-\left(\frac{1}{2}\times 1+\frac{3}{2}\times 1\right)=16$	A1
		(6)
7 (ii)	$\frac{6x^2-16}{(x+1)(2x-3)}=3+\ldots..$	B1
	$\frac{6x^2-16}{(x+1)(2x-3)}=3+\frac{2}{x+1}-\frac{1}{2x-3}$	M1 A1
	$\int \frac{6x^2-16}{(x+1)(2x-3)}\,\mathrm{d}x=\int 3+\frac{2}{x+1}-\frac{1}{2x-3}\,\mathrm{d}x$	
	$=3x+2\ln\lvert x+1\rvert-\frac{1}{2}\ln\lvert 2x-3\rvert+c$	M1 A1ft A1
		(6)
		(12 marks)

(i)

B1: States or uses a suitable substitution. That is one that will work.

M1: Attempts to change the integral in x to an integral in 'u'

Expect to see all aspects changed including the dx

For $u=\sqrt{2x-1}$ expect to see $\int Au^2+B\,\mathrm{d}u$

For $u = 2x-1$ expect to see $\int Pu^{\frac{1}{2}} + Qu^{-\frac{1}{2}} \mathrm{d}u$

dM1: For correct method of integration. Allow for correct powers. Dependent upon previous M

A1: See scheme. Correct integration which may be left unsimplified

M1: Uses correct limits for their **valid** substitution.

A1: CSO 16. Look for 18 -2 or such before the 16.

16 with no working scores no marks. This is evidence of use of a calculator

(ii) Note that epen scoring is M1 M1 A1 M1 A1 A1

B1: For a quotient of 3.

This can be scored by setting $6x^2 - 16 = A(x+1)(2x-3) + B(x+1) + C(2x-3)$ and reaching $A = 3$

or by division with 3 in the "correct place"

M1: A full attempt to find A, B and C.

Either sets $6x^2 - 16 = A(x+1)(2x-3) + B(x+1) + C(2x-3)$ condoning slips leading to values of A, B and C

or divides and uses partial fractions on the remainder to form an expression of the correct form.

A1: Correct partial fractions seen or implied. $\dfrac{6x^2-16}{(x+1)(2x-3)} = 3 + \dfrac{2}{(x+1)} - \dfrac{1}{(2x-3)}$

M1: Two of $A \to ...x$, $\displaystyle\int \frac{B}{(x+1)}\,\mathrm{d}x \to ...\ln(x+1)$ $\displaystyle\int \frac{C}{(2x-3)}\,\mathrm{d}x \to ...\ln(2x-3)$

A1ft: Two correct of $A \to Ax$ $\displaystyle\int \frac{B}{(x+1)}\,\mathrm{d}x \to B\ln(x+1)$ and $\displaystyle\int \frac{C}{(2x-3)}\,\mathrm{d}x \to \frac{C}{2}\ln(2x-3)$

Please note that answers such as $\displaystyle\int \frac{1}{2(2x-3)}\,\mathrm{d}x \to \frac{1}{2}\ln(4x-6)$ are correct

A1: $3x + 2\ln|x+1| - \dfrac{1}{2}\ln|2x-3| + c$ but condone $3x + 2\ln(x+1) - \dfrac{1}{2}\ln(2x-3) + c$ **The $+c$ is required.**

You should isw after a correct answer.

Note that it is possible to do **part (i)** by parts. This does not satisfy the demand of the question but can score some marks

B1: $\displaystyle\int \frac{3x}{\sqrt{2x-1}}\,\mathrm{d}x = 3x\sqrt{2x-1} - \int 3\sqrt{2x-1}\,\mathrm{d}x$

M1: $\displaystyle\int \frac{3x}{\sqrt{2x-1}}\,\mathrm{d}x = ax\sqrt{2x-1} - b(2x-1)^{\frac{3}{2}}$ FYI: Correct when $a = 3, b = 1$

third M1 (5th mark on epen): applies limits to a valid expression $\displaystyle\int_1^5 \frac{3x}{\sqrt{2x-1}}\,\mathrm{d}x = \left[ax\sqrt{2x-1} - b(2x-1)^{\frac{3}{2}}\right]_1^5 = ...$

second A1 (6th mark on epen) $\displaystyle\int_1^5 \frac{3x}{\sqrt{2x-1}}\,\mathrm{d}x = \left[3x\sqrt{2x-1} - (2x-1)^{\frac{3}{2}}\right]_1^5 = (45-27) - (3-1) = 16$ o.e.

to score on e-epen 110011

Note that in **part (ii)** some candidates will write

$\frac{6x^2-16}{(x+1)(2x-3)}=\frac{B}{x+1}+\frac{C}{2x-3}$ and via substitution will get "a correct" $\frac{2}{x+1}-\frac{1}{2x-3}$

$\int\frac{6x^2-16}{(x+1)(2x-3)}\,dx=\int\frac{"B"}{x+1}+\frac{"C"}{2x-3}\,dx="B"\ln(x+1)+\frac{"C"}{2}\ln(2x-3)$

Scores SC B0 M0 A0 M1 A1ft A0 for 2 out of 6 if ft integration is correct

Question Number	Scheme	Marks
8 (a)	$\begin{pmatrix}4\\-3\\2\end{pmatrix}+\lambda\begin{pmatrix}3\\-2\\-1\end{pmatrix}=\begin{pmatrix}2\\0\\-9\end{pmatrix}+\mu\begin{pmatrix}2\\-1\\-3\end{pmatrix}\Rightarrow\begin{matrix}4+3\lambda=2+2\mu\\-3-2\lambda=0-1\mu\\2-1\lambda=-9-3\mu\end{matrix}$ **any two** of these	M1
	Full method to find either λ **or** μ eg $(1)+3(3)\Rightarrow\mu=...$	M1
	Either $\lambda=-4$ or $\mu=-5$	A1
	Position vector of intersection is $\begin{pmatrix}4\\-3\\2\end{pmatrix}-4\begin{pmatrix}3\\-2\\-1\end{pmatrix}=$ OR $\begin{pmatrix}2\\0\\-9\end{pmatrix}-5\begin{pmatrix}2\\-1\\-3\end{pmatrix}=$	dM1
	$=\begin{pmatrix}-8\\5\\6\end{pmatrix}$	A1 **(5)**
(b)	Co-ordinates or position vector of point $Q=\begin{pmatrix}2+2\mu\\0-1\mu\\-9-3\mu\end{pmatrix}$	
	$\overrightarrow{PQ}=\begin{pmatrix}2+2\mu\\0-1\mu\\-9-3\mu\end{pmatrix}-\begin{pmatrix}10\\-7\\0\end{pmatrix}=\begin{pmatrix}2\mu-8\\7-1\mu\\-9-3\mu\end{pmatrix}$	M1
	Uses $\overrightarrow{PQ}.\begin{pmatrix}2\\-1\\-3\end{pmatrix}=0\Rightarrow4\mu-16-7+\mu+27+9\mu=0\Rightarrow\mu=-\frac{2}{7}$	dM1 A1
	Substitutes their μ into $\begin{pmatrix}2+2\mu\\0-1\mu\\-9-3\mu\end{pmatrix}\Rightarrow Q=\left(\frac{10}{7},\frac{2}{7},-\frac{57}{7}\right)$	ddM1 A1 **(5)** **(10 marks)**