7.

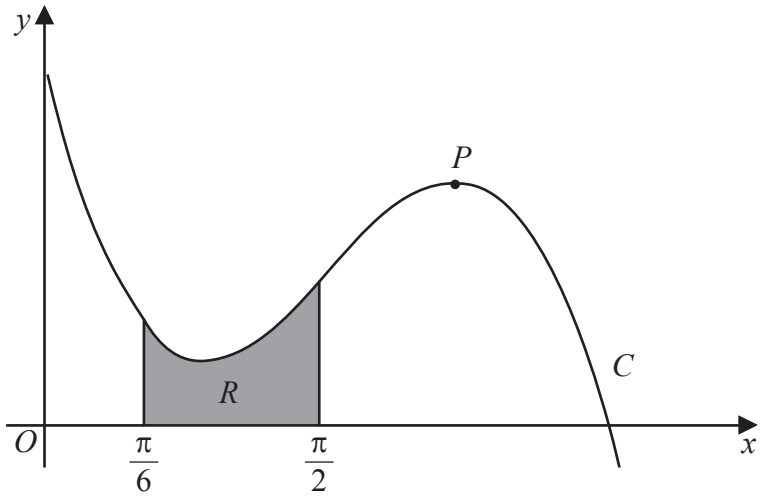

Figure 3

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 3 shows a sketch of part of the curve C with equation

$$y = 1 - 2x + 5\sin^2\left(\frac{1}{2}x\right) \qquad x > 0$$

where x is measured in radians.

The point P lies on C and is a local maximum, as shown in Figure 3.

(a) Use calculus and a suitable trigonometric identity to find the x coordinate of P.
Give the answer to 3 significant figures.

(4)

The region R, shown shaded in Figure 3, is bounded by C, the x-axis and the lines with equations $x = \frac{\pi}{6}$ and $x = \frac{\pi}{2}$

(b) Use algebraic integration to find the exact area of R.

(4)

(Total for Question 7 is 8 marks)

8. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The number of electric cars and the number of petrol cars being registered in a country were monitored over the same period of time.

The number of electric cars, E thousands, registered in the country t years after monitoring began, is modelled by the equation

$$E = A + 40e^{0.2t} \qquad t \geqslant 0$$

where A is a constant.

Given that $E = 5$ when $t = 0$

(a) state the value of A.

(1)

Use the equation for this model to answer parts (b) and (c).

(b) Find the number of electric cars registered in the country when $t = 3$, giving the answer to 3 significant figures.

(2)

(c) Find the **rate of increase** in the number of electric cars registered in the country when $t = 5$, giving the answer to 3 significant figures.

(2)

The number of petrol cars, P thousands, registered in the country t years after monitoring began, is modelled by the equation

$$P = 150 + 450e^{-0.2t} \qquad t \geqslant 0$$

When $t = X$, the number of electric cars registered in the country was equal to the number of petrol cars registered in the country.

Using the equations of both models,

(d) find the value of X, giving the answer to one decimal place.

(*You must show sufficient algebraic working to make your method clear.*)

(4)

(Total for Question 8 is 9 marks)