Question Number	Scheme	Notes	Marks
4	**"In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable."**		
	$(\sin 3x+\cos 3x)^2=\sin^2 3x+\cos^2 3x+2\sin 3x\cos 3x=1+2\sin 3x\cos 3x=1+\sin 6x$ Expands correctly and obtains an appropriate clearly integrable function: $(\sin 3x+\cos 3x)^2\rightarrow\pm1\pm\sin 6x$ (identity sign errors only). See variation below and Alt		**M1**
	$\int_{\frac{\pi}{18}}^{\frac{\pi}{12}}1+\sin 6x\ \mathrm{d}x=\left[x-\frac{\cos 6x}{6}\right]_{\frac{\pi}{18}}^{\frac{\pi}{12}}$	**d**M1: Integrates $\pm1\pm\sin 6x$ to $\pm x\pm r\cos 6x$ A1: $x-\frac{\cos 6x}{6}$ Must not be fortuitous. May have done this with more than one integral	**dM1** **A1**
	$\left(\frac{\pi}{12}-\frac{\cos\left(\frac{\pi}{2}\right)}{6}\right)-\left(\frac{\pi}{18}-\frac{\cos\left(\frac{\pi}{3}\right)}{6}\right)=\frac{\pi}{12}-\frac{\pi}{18}-0+\frac{1}{12}=\frac{\pi}{36}+\frac{1}{12}$ M1: Substitutes in the correct limits following an attempt at integration and subtracts either way round to obtain a two term expression of the form $a\pi+b,\ a,b\neq 0$ and b may be decimal. Allow slips if intention clear. If there is a substitution e.g., $u=6x$ and they do not revert to x then the changed limits must be correct e.g., $\frac{\pi}{2}$ and $\frac{\pi}{3}$ A1: Correct exact two term numerical expression. May be factorised (Isw) Must not be fortuitous.		**M1** **A1**
	An unsimplified variation on the above is to reach $\frac{1-\cos 6x}{2}+\frac{1+\cos 6x}{2}+\sin 6x$ (M1: identity sign errors only) $\Rightarrow\frac{1}{2}x-\frac{\sin 6x}{12}+\frac{1}{2}x+\frac{\sin 6x}{12}-\frac{\cos 6x}{6}$ (**d**M1A1) For the **d**M1 they must be integrating an expression that is equivalent to $\pm1\pm\sin 6x$ and they must achieve an expression that is equivalent to $\pm x\pm r\cos 6x$. Final M1 and A1 as before noting that the limits may be applied to separate terms		
			(Total 5 marks)
Alt	$(\sin 3x+\cos 3x)^2=\sin^2 3x+\cos^2 3x+2\sin 3x\cos 3x=1+2\sin 3x\cos 3x$ $\int_{\frac{\pi}{18}}^{\frac{\pi}{12}}1+2\sin 3x\cos 3x\ \mathrm{d}x=\left[x+\frac{\sin^2 3x}{3}\right]_{\frac{\pi}{18}}^{\frac{\pi}{12}}$ or $\left[x-\frac{\cos^2 3x}{3}\right]_{\frac{\pi}{18}}^{\frac{\pi}{12}}$ M1**d**M1: Obtains $\pm1\pm2\sin 3x\cos 3x$ and integrates to $\pm x\pm A\sin^2 3x$ or $\pm x\pm B\cos^2 3x$ The two M marks are awarded together here. Note that they could also use $\sin^2 3x+\cos^2 3x=\frac{1-\cos 6x}{2}+\frac{1+\cos 6x}{2}$ as in the main scheme variation A1: Correct integration		**M1** **dM1** **A1**
	$\left(\frac{\pi}{12}+\frac{\sin^2\left(\frac{\pi}{4}\right)}{3}\right)-\left(\frac{\pi}{18}+\frac{\sin^2\left(\frac{\pi}{6}\right)}{3}\right)$ or $\left(\frac{\pi}{12}-\frac{\cos^2\left(\frac{\pi}{4}\right)}{3}\right)-\left(\frac{\pi}{18}-\frac{\cos^2\left(\frac{\pi}{6}\right)}{3}\right)=....\qquad\frac{\pi}{36}+\frac{1}{12}$ M1: Substitutes in the limits following an attempt at integration and subtracts either way round to reach $a\pi+b,\ a,b\in\mathbb{R}$. Trig evaluated. b may be irrational. Allow slips if intention clear.		**M1** **A1**

	A1: Correct exact two term numerical expression. May be factorised. Isw if necessary	
Note	If an alternative method of integration is used score the first M for reaching a clearly integrable function, **d**M1 for achieving a correct form after integration and A1 if completely correct. Allow sign errors only with identities used.	

Question Number	Scheme	Notes	Marks
5	$H = ab^t \Rightarrow \log_{10} H = \log_{10} a + (\log_{10} b)t$		
(a)	$\log_{10} a = 2.5$ **or** $\log_{10} b = \dfrac{3.25-2.5}{10-0}$ $\left\{= \dfrac{3}{40},\ 0.075\right\}$	One correct equation in any form. Implied by appropriate expressions/values. The base of 10 may be implied later and allow e.g., lg a	**M1**
	$a = 10^{2.5}$ or $10^{\frac{5}{2}}$ or awrt 316 $b = 10^{\frac{3.25-2.5}{10-0}}$ or $10^{0.075}$ or $10^{\frac{3}{40}}$ or awrt 1.19	Any correct expression (or value) for a or b	**A1**
	$\log_{10} a = 2.5$ **and** $\log_{10} b = \dfrac{3.25-2.5}{10-0}$ $\left\{= \dfrac{3}{40},\ 0.075\right\}$	Both correct equations in any form. Implied by appropriate expressions/values. The bases of 10 may be implied later	**dM1**
	Allow the above marks to be implied by the equation $H = 10^{\frac{5}{2}} \times 10^{\frac{3}{40}t}$ **but** $H = 10^{\frac{5}{2}+\frac{3}{40}t}$ or $H = 10^{2.5+0.075t}$ or $\log_{10} H = \dfrac{3}{40}t + 2.5$ need further work		
	$a = $ awrt 316, $b = $ awrt 1.19 Note: $a = 316.22776...$, $b = 1.18850...$	Both correct **decimal** values to **3sf** Answers only are acceptable. Accept values just embedded in the **given** model i.e., $H = 316 \times 1.19^t$ or e.g., $(1.19)^t .316$	**A1**
			(4)
(b)	e.g. {the number of households recycling their household waste} is **<u>increasing</u>** by **<u>19%</u>** **<u>per year</u>** **oe** ft their b from part (a) provided $b > 1$ and percentage correct to 2sf All 3 underlined parts are required but accept surrogates such as rising/going up....by a rate/factor/multiplier of "1.19"...year on year/each year/per annum Do not accept: The amount it is rising/The rate by which it increases/The percentage increase each year/The annual rate of increase/It is **changing** by 19% per year/The annual increase is 119%		**B1ft**
			(1)
(c)	*Solutions relying entirely on calculator technology are not acceptable.*		
	$\left\{\dfrac{\mathrm{d}H}{\mathrm{d}t}\right\} = 316 \times 1.19^t \times \ln(1.19),\ \Rightarrow \left\{\dfrac{\mathrm{d}H}{\mathrm{d}t}_{t=5} =\right\}$ awrt 130 M1: Differentiates to the form $ab^t(\ln b)$ oe - **correct** for their positive constants. Not e.g., "log" unless ln is later implied. Allow this mark with "a" and "b" instead of values. $t = 5$ may be already substituted and allow if it is partially calculated provided there is at least a product of two numbers seen (and those numbers are correct to 2sf) and it is correct for their values. A minimum would be e.g., 316×0.415		**M1** **A1**

	A1: Obtains awrt 130 (**2 sf**). Not ft Answer only of 130 (131.176059... or 129.50219... (accurate)) is 00	
		(2)
		(Total 7 marks)