1. A curve has equation $y = f(x)$ where

$$f(x) = \frac{2}{3}x^3 - 3x^2 + 5x + e^{2x}(1 - 2x) \qquad x \in \mathbb{R}$$

(a) Find $f'(x)$ giving the answer in simplest form.

(3)

The equation $f'(x) = 0$ has a single root, α

(b) Show that α lies in the interval $[0, 1]$

(2)

Given that the equation $f'(x) = 0$ can be written as

$$x = \frac{2x^2 + 5}{4e^{2x} + 6}$$

(c) use the iteration formula

$$x_{n+1} = \frac{2x_n^{\,2} + 5}{4e^{2x_n} + 6} \qquad \text{with } x_1 = 0$$

to find, to 4 decimal places,

(i) the value of x_3

(ii) the value of α

(3)

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Question 1 continued

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Question 1 continued

(Total for Question 1 is 8 marks)

2. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The functions f and g are defined by

$$f(x) = 3 + 4e^{2x} \qquad x \in \mathbb{R}$$

$$g(x) = \sqrt{x + 5} \qquad x \in \mathbb{R} \quad x > -5$$

(a) State the range of f

(1)

(b) Find g^{-1}

(3)

(c) Find $gf\left(\ln\sqrt{7}\right)$

(2)

(d) Find the value of a, $a > 0$, for which

$$fg(a^2 - 5) = g(59)$$

giving the answer in the form $\ln p$ where p is an irrational constant.

(3)

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