Question Number	Scheme	Marks
6 (a)	$1000 < V \leqslant 23000$	B1,B1
		(2)
(b)	$\dfrac{\mathrm{d}V}{\mathrm{d}t} = 18000 \times -0.2\mathrm{e}^{-0.2t} + 4000 \times -0.1\mathrm{e}^{-0.1t}$	M1
	$\left.\dfrac{\mathrm{d}V}{\mathrm{d}t}\right\|_{t=10} = 18000 \times -0.2\mathrm{e}^{-2} + 4000 \times -0.1\mathrm{e}^{-1} = (-)634\,(\text{awrt})$	dM1A1
		(3)
(c)	$(15000 = 18000\mathrm{e}^{-0.2t} + 4000\mathrm{e}^{-0.1t} + 1000 \Rightarrow 0 = 9\mathrm{e}^{-0.2t} + 2\mathrm{e}^{-0.1t} - 7)$	
	$0 = (9\mathrm{e}^{-0.1t} - 7)(\mathrm{e}^{-0.1t} + 1)$	M1A1
	$9\mathrm{e}^{-0.1t} = 7 \Rightarrow t = 10\ln\left(\dfrac{9}{7}\right)\text{oe}$	dM1A1
		(4)
		(9 marks)

Notes

(a)

B1: Achieves the value of one end, either 23000 or 1000 associated to the correct end. Condone for this mark the incorrect omission or inclusion of the boundary. Eg $V < 23000$ but do not condone $V \geq 23000$ or just 23000 with no inequality (or similar with the 1000).

B1: Completely correct solution. Accept $1000 < V \leq 23000$, $1000 < \mathit{Range} \leq 23000$, (1000, 23000]. The inequalities must be correct, $1000 < V < 23000$ is B0.

(b)

M1: Differentiates to form $\dfrac{\mathrm{d}V}{\mathrm{d}t} = A\mathrm{e}^{-0.2t} + B\mathrm{e}^{-0.1t} + (0)$ where $A \neq 18000$, $B \neq 4000$

dM1: Substitutes $t = 10$ into their $\dfrac{\mathrm{d}V}{\mathrm{d}t}$. It is dependent upon having scored the previous M1.

A1: Correct solution only. Accept awrt ± 634 following a correct $\dfrac{\mathrm{d}V}{\mathrm{d}t} = -3600\mathrm{e}^{-0.2t} - 400\mathrm{e}^{-0.1t}$. Do not isw if they change to 6.34 as their final answer.

Watch for students who substitute $t = 10$ into their V first and then differentiate. This is M0dM0A0.

Watch for students who achieve +634 following $\dfrac{\mathrm{d}V}{\mathrm{d}t} = 3600\mathrm{e}^{-0.2t} + 400\mathrm{e}^{-0.1t}$. This is M1dM1A0

(c)

M1: Sets up a 3TQ in $\mathrm{e}^{\pm 0.1t}$ from the information **AND** makes a valid attempt to factorise or solve the quadratic. May be implied by reaching $\mathrm{e}^{\pm 0.1t} = \dfrac{7}{9}$ or $\dfrac{9}{7}$ or root of their quadratic either way up. May use a subsitution and solve a quadratic in their variable.

A1: Correct factors $(9\mathrm{e}^{-0.1t} - 7)(\mathrm{e}^{-0.1t} + 1)$ or $(7\mathrm{e}^{0.1t} - 9)(\mathrm{e}^{0.1t} + 1)$ or correct solutions associated to the correct exponential terms, though they may omit the negative solution.

dM1: Dependent upon the previous M1. Proceeds via correct ln work to find the exact value for t

E.g. This is scored for setting the $a\mathrm{e}^{\pm 0.1t} - b = 0$ and proceeding 'correctly' to $t = ...$ or if quadratic solved by other means it is for proceeding from $\mathrm{e}^{\pm 0.1t} = K$ correctly to achieve a value for t.

A1: $t = 10\ln\left(\dfrac{9}{7}\right)$. Accept other simplest form alternatives such as $-10\ln\left(\dfrac{7}{9}\right)$ or $\ln\left(\dfrac{9}{7}\right)^{10}$ and withhold if any extra solutions are given. ISW a correct answer as long as no extra solutions are given.

NB Answers only will score no marks in part (c).

Question Number	Scheme	Marks
7.	$\sqrt{5}\cos\theta - 2\sin\theta \equiv R\cos(\theta + \alpha)$	
(a)	$R = 3$	B1
	$\tan\alpha = \dfrac{2}{\sqrt{5}}, \Rightarrow \alpha = \text{awrt } 0.7297 \text{ (4 sf)}$	M1, A1
		(3)
(b)	$3\cos(\theta + 0.7297) = 0.5 \Rightarrow \cos(\theta + 0.7297) = \dfrac{0.5}{3}$	M1
	$\Rightarrow \theta_1 = 0.674 \text{ (3 sf)}$	A1
	$\theta_2 + \text{"0.7297"} = \text{"}-1.4033\text{"} \Rightarrow \theta_2 = \ldots$	dM1
	$\Rightarrow \theta_2 = -2.13 \text{ (3 sf)}$	A1
		(4)
(c)	E.g. $\begin{matrix}3A + B = 33 \\ -3A + B = -15\end{matrix}$ or $\begin{matrix}3A + B = -15 \\ -3A + B = 33\end{matrix}$ or $B = \dfrac{33 - 15}{2} = \ldots$ $\Rightarrow A = \ldots$ or $B = \ldots$	M1
	$A = 8, B = 9$ and $A = -8, B = 9$	A1A1A1
		(4)
		(11 marks)

Notes

(a)

B1: For $R = 3$ stated or seen in their expression. $\sqrt{9}$ is B0. ± 3 is B0.

M1: $\tan\alpha = \pm\dfrac{2}{\sqrt{5}}$, $\tan\alpha = \pm\dfrac{\sqrt{5}}{2} \Rightarrow \alpha = \ldots$

Also allow $\cos\alpha = \pm\dfrac{\sqrt{5}}{3}$ or $\pm\dfrac{2}{3}$, $\sin\alpha = \pm\dfrac{2}{3}$ or $\pm\dfrac{\sqrt{5}}{3} \Rightarrow \alpha = \ldots$, where "3" is their R

Note the M may be scored if they find α in degrees (41.8°).

A1: $\alpha = \text{awrt } 0.7297 \text{ (4 sf)}$. No need to state the full expression for this mark.

(b)

M1: Uses part (a) and proceeds to $\cos(\theta + \text{"0.7297"}) = \dfrac{0.5}{\text{"3"}}$. Must see some working, but may be implied by $\theta \pm \text{"0.7297"} = 1.4033$ or $\theta \pm \text{"0.7297"} = \cos^{-1}\left(\dfrac{0.5}{\text{their 3}}\right)$

A1: Achieves a correct value for θ of awrt 0.674

dM1: Correct method for a second angle, usually given for θ_2 + their 0.7297 = − their 1.4033 $\Rightarrow \theta_2 = \ldots$

May be implied but the previous M must have been gained. If working in degrees as long as they are consistently in degrees.

A1: $\Rightarrow \theta_2 = \text{awrt} -2.13 \text{ (3 sf)}$ but withhold if there are extra solutions in the range and first A scored.

For candidates who work consistently in degrees in (a) and (b) allow awrt 38.6° for the first A in (b) or awrt – 122° for the second A mark if the first A was lost due to error (i.e. max one A mark).

(c)

M1: Correct method to find at least one value. E.g. writes down at least one pair of simultaneous equations (or in-equations) of form $-15 = kA + B$, $33 = -kA + B$ or uses B is mean value of end points other suitable working or equation and finds at least one of $A = \ldots$ or $B =$

Note correct value for A or B implies M1A1 (with no incorrect working for it)

A1: At least one of $A = 8$, $B = 9$ or $A = -8$

A1: At least two of $A = 8$, $B = 9$ or $A = -8$

A1: For all three of $A = 8$, $B = 9$ and $A = -8$ and no other values.