Question Number	Scheme	Marks
5 (a)	Way 1: $\dfrac{\cot^2 x}{1+\cot^2 x} \equiv \dfrac{\cos^2 x \big/ \sin^2 x}{\operatorname{cosec}^2 x} \equiv \dfrac{\cos^2 x \big/ \sin^2 x}{1 \big/ \sin^2 x} \equiv \cos^2 x$	M1, M1, A1
	Way 2: $\dfrac{\cot^2 x}{1+\cot^2 x} \equiv \dfrac{\cos^2 x}{\sin^2 x+\cos^2 x} \equiv \dfrac{\cos^2 x}{1} \equiv \cos^2 x$	M1, M1 A1
	Way 3: Consider $\cos^2 x(1+\cot^2 x) = \cos^2 x \operatorname{cosec}^2 x = \dfrac{\cos^2 x}{\sin^2 x} = \cot^2 x$ Hence $\dfrac{\cot^2 x}{1+\cot^2 x} \equiv \cos^2 x$	M1 M1 A1
	Way 4: Consider $\cos^2 x(1+\cot^2 x) = \cos^2 x(1+\dfrac{\cos^2 x}{\sin^2 x}) = \dfrac{\cos^2 x(\sin^2 x+\cos^2 x)}{\sin^2 x} = \dfrac{\cos^2 x}{\sin^2 x} = \cot^2 x$ Hence $\dfrac{\cot^2 x}{1+\cot^2 x} \equiv \cos^2 x$	M1 M1 A1
		(3)
(b)	$\cos^2 x = 8(2\cos^2 x - 1) + 2\cos x$	M1
	$15\cos^2 x + 2\cos x - 8 = 0$	A1
	So cos$x$ = 2/3 or $-4/5$ leading to at least one value for x	dM1
	$\Rightarrow x = 48.2°$ or $143.1°$ or $216.9°$ or $311.8°$	A1, A1
		(5)
		(8 marks)

Notes

(a)

Way 1 and Way 3:

M1: See one of $1+\cot^2 x = \operatorname{cosec}^2 x$, $\cot^2 x = \cos^2 x \big/ \sin^2 x$ used.

M1: See two of $1+\cot^2 x = \operatorname{cosec}^2 x$, $\cot^2 x = \cos^2 x \big/ \sin^2 x$, $\operatorname{cosec}^2 x = 1 \big/ \sin^2 x$ used.

A1: See all three of $1+\cot^2 x = \operatorname{cosec}^2 x$, $\cot^2 x = \cos^2 x \big/ \sin^2 x$, $\operatorname{cosec}^2 x = 1 \big/ \sin^2 x$ used and **conclusion reached** – which may be reaching $\cos^2 x$ if working from LHS to RHS.

Way 2:

M1: See one of $\sin^2 x(1+\cot^2 x) = \sin^2 x + \cos^2 x$, $\cot^2 x \sin^2 x = \cos^2 x$ used **or implied**

M1: See two of $\sin^2 x(1+\cot^2 x) = \sin^2 x + \cos^2 x$, $\cot^2 x \sin^2 x = \cos^2 x$, $\sin^2 x + \cos^2 x = 1$ used **or implied**

A1: See all three of $\sin^2 x(1+\cot^2 x) = \sin^2 x + \cos^2 x$, $\cot^2 x \sin^2 x = \cos^2 x$, $\sin^2 x + \cos^2 x = 1$ used **or implied** and **conclusion reached**. In this method multiplication of numerator and denominator may be implied and $\sin^2 x(1+\cot^2 x) = \sin^2 x + \cos^2 x$ **may not be explicit**.

Way 4:

M1: See $\cot^2 x = \cos^2 x \big/ \sin^2 x$ used

M1: See both $\cot^2 x = \cos^2 x \big/ \sin^2 x$ and $(1+\dfrac{\cos^2 x}{\sin^2 x}) = \dfrac{(\sin^2 x+\cos^2 x)}{\sin^2 x}$ used

A1: See all three of $\cot^2 x = \cos^2 x \big/ \sin^2 x$ $(1+\dfrac{\cos^2 x}{\sin^2 x}) = \dfrac{(\sin^2 x+\cos^2 x)}{\sin^2 x}$ and $\sin^2 x + \cos^2 x = 1$ used and **conclusion reached**.

Note: for conclusions, if working from one side direct to the other, accept successfully reaching the other side as the conclusion. If multiplying or not starting directly from one side some concluding statement will be required (e.g. "hence proved").

There will be some combinations of these methods or variations on these. **A complete method with no errors scores** M1M1A1

In general score M1 for one valid and appropriate identity used (possibly implied), M1 for two valid and appropriate (different) identities applied within a single proof and A1 for a complete process using all necessary identities and if not proving from one side to the other directly a conclusion will also be needed.

E,g. $\dfrac{\cot^2 x}{1+\cot^2 x} = \dfrac{\cos^2 x \big/ \sin^2 x}{\operatorname{cosec}^2 x} = \dfrac{\cos^2 x}{\sin^2 x \operatorname{cosec}^2 x} = \cos^2 x$ scores M1 for use of $\cot^2 x = \dfrac{\cos^2 x}{\sin^2 x}$, M1 for use of $1+\cot^2 x = \operatorname{cosec}^2 x$ (or vice versa) and A for the complete proof with all appropriate identities used.

Note: Condone poor notation for the M marks, but for the final A the notation should be correct throughout the proof, allowing a single notational slip if recovered.

(b)

M1: Uses part (a) to replace left hand side and uses double angle formula for cos 2x on right hand side to get an equation in cos$x$ only.

Condone $\cos 2x = \pm 2\cos^2 x \pm 1$ for the double angle identity. If cos 2x = $\cos^2 x - \sin^2 x$ is used this mark is not scored until the $\sin^2 x$ is replaced by $1 - \cos^2 x$. Condone a slip or an omission on either of the coefficients 8 and 2.

A1: Correct three term quadratic reached. Terms should all be gathered on one side unless an appropriate completing the square method is shown to solve to imply it. The = 0 may be implied.

dM1: Solves their 3TQ in cos$x$ by any method – factorising, formula or completion of square or just writing down answers (must be correct for their quadratic if no method shown – you may need to check - an invalid solutions for cos$x$ may be omitted). You must see a value for cos$x$ (or e.g. value for a if as substitution a = cos$x$ has been used) appearing before sight of a value for x. No value for cos x seen will be dM0A0A0 (even if correct answer appear).

A1: Two correct answers in the range. Must be in degrees.

A1: All four answers in the range, $x = awrt\ 48.2°$ or $143.1°$ or $216.9°$ or $311.8°$

Any extra solutions in the range withhold the last A mark.

Ignore any solutions outside the range $0 \leqslant x \leqslant 360°$

Question Number	Scheme	Marks
6 (a)	$1000 < V \leqslant 23000$	B1,B1
		(2)
(b)	$\dfrac{\mathrm{d}V}{\mathrm{d}t} = 18000 \times -0.2\mathrm{e}^{-0.2t} + 4000 \times -0.1\mathrm{e}^{-0.1t}$	M1
	$\left.\dfrac{\mathrm{d}V}{\mathrm{d}t}\right\|_{t=10} = 18000 \times -0.2\mathrm{e}^{-2} + 4000 \times -0.1\mathrm{e}^{-1} = (-)634\,(\text{awrt})$	dM1A1
		(3)
(c)	$(15000 = 18000\mathrm{e}^{-0.2t} + 4000\mathrm{e}^{-0.1t} + 1000 \Rightarrow 0 = 9\mathrm{e}^{-0.2t} + 2\mathrm{e}^{-0.1t} - 7)$	
	$0 = (9\mathrm{e}^{-0.1t} - 7)(\mathrm{e}^{-0.1t} + 1)$	M1A1
	$9\mathrm{e}^{-0.1t} = 7 \Rightarrow t = 10\ln\left(\dfrac{9}{7}\right)\text{oe}$	dM1A1
		(4)
		(9 marks)

Notes

(a)

B1: Achieves the value of one end, either 23000 or 1000 associated to the correct end. Condone for this mark the incorrect omission or inclusion of the boundary. Eg $V < 23000$ but do not condone $V \geq 23000$ or just 23000 with no inequality (or similar with the 1000).

B1: Completely correct solution. Accept $1000 < V \leq 23000$, $1000 < \mathit{Range} \leq 23000$, (1000, 23000]. The inequalities must be correct, $1000 < V < 23000$ is B0.

(b)

M1: Differentiates to form $\dfrac{\mathrm{d}V}{\mathrm{d}t} = A\mathrm{e}^{-0.2t} + B\mathrm{e}^{-0.1t} + (0)$ where $A \neq 18000$, $B \neq 4000$

dM1: Substitutes $t = 10$ into their $\dfrac{\mathrm{d}V}{\mathrm{d}t}$. It is dependent upon having scored the previous M1.

A1: Correct solution only. Accept awrt ± 634 following a correct $\dfrac{\mathrm{d}V}{\mathrm{d}t} = -3600\mathrm{e}^{-0.2t} - 400\mathrm{e}^{-0.1t}$. Do not isw if they change to 6.34 as their final answer.

Watch for students who substitute $t = 10$ into their V first and then differentiate. This is M0dM0A0.

Watch for students who achieve +634 following $\dfrac{\mathrm{d}V}{\mathrm{d}t} = 3600\mathrm{e}^{-0.2t} + 400\mathrm{e}^{-0.1t}$. This is M1dM1A0

(c)

M1: Sets up a 3TQ in $\mathrm{e}^{\pm 0.1t}$ from the information **AND** makes a valid attempt to factorise or solve the quadratic. May be implied by reaching $\mathrm{e}^{\pm 0.1t} = \dfrac{7}{9}$ or $\dfrac{9}{7}$ or root of their quadratic either way up. May use a subsitution and solve a quadratic in their variable.

A1: Correct factors $(9\mathrm{e}^{-0.1t} - 7)(\mathrm{e}^{-0.1t} + 1)$ or $(7\mathrm{e}^{0.1t} - 9)(\mathrm{e}^{0.1t} + 1)$ or correct solutions associated to the correct exponential terms, though they may omit the negative solution.

dM1: Dependent upon the previous M1. Proceeds via correct ln work to find the exact value for t

E.g. This is scored for setting the $a\mathrm{e}^{\pm 0.1t} - b = 0$ and proceeding 'correctly' to $t = ...$ or if quadratic solved by other means it is for proceeding from $\mathrm{e}^{\pm 0.1t} = K$ correctly to achieve a value for t.

A1: $t = 10\ln\left(\dfrac{9}{7}\right)$. Accept other simplest form alternatives such as $-10\ln\left(\dfrac{7}{9}\right)$ or $\ln\left(\dfrac{9}{7}\right)^{10}$ and withhold if any extra solutions are given. ISW a correct answer as long as no extra solutions are given.

NB Answers only will score no marks in part (c).