Question Number	Scheme		Marks
4	Differentiates wrt x $\quad \frac{dy}{dx} = 16\sec^2(2x)$	Gives $x = \frac{1}{2}\arctan\left(\frac{y}{8}\right)$	M1
	Inverts to get $\frac{dx}{dy} = \frac{1}{16\sec^2 2x} = \frac{1}{16(1+\tan^2 2x)}$ Or $\frac{dy}{dx} = 16\left(1+\tan^2 2x\right) = 16\times\left(1+\left(\frac{y}{8}\right)^2\right)$	$\frac{dx}{dy} = K \times \frac{1}{\left(1+\left(\frac{y}{8}\right)^2\right)}$	dM1
	$\frac{dx}{dy} = \frac{A}{B+y^2}$	$\frac{dx}{dy} = \frac{1}{2}\cdot\frac{1/8}{(1+\left(\frac{y}{8}\right)^2)} = \frac{A}{B+y^2}$	ddM1
	$= \frac{4}{64+y^2}$		A1
			(4)
			(4 marks)

Notes

Way 1:

M1: Achieves $\left(\frac{dy}{dx}\right) = \lambda\sec^2(2x)$ or $1 = \lambda\sec^2(2x)\frac{dx}{dy}$ (oe) In the second case the $\frac{dx}{dy}$ must be present, but for this mark allow if just "$=\lambda\sec^2 2x$" is given – but the $\frac{dy}{dx}$ must be present for further marks to be scored.

If they change tan 2x to $\frac{\sin 2x}{\cos 2x}$ they can score this mark for $\frac{dy}{dx} = \frac{a\cos 2x\cos 2x \pm b\sin 2x\sin 2x}{(\cos 2x)^2}$

If other identities are seen progress is unlikely and must reach a derivative in terms of trig terms in 2x.

For instance, use of $y = \frac{16\tan x}{1-\tan^2 x}$ will not like make suitable porgress as they will be working in tan$x$. Only if suitable identities to get in to 2x terms is made will they become elligible for this mark. Review any cases you feel worthy of merit and do not know how to score.

dM1: Either

- finds reciprocal of their function (or makes $\frac{dx}{dy}$ the subject) **and** uses $1+\tan^2 2x = \sec^2 2x$

OR

- uses $1+\tan^2 2x = \sec^2 2x$ **and** replaces tan 2x by y/8 to get and expression in y only

ddM1: Full process (all three steps noted in the dM mark) to make $\frac{dx}{dy}$ the subject in terms of y and eliminate fractions to reach the correct form. Note, correct replacement of tan2x by y/8 is required but there may be algebraic slips following that in simplifying to the correct form.

A1: cao and depends on all previous marks.

Way 2:

M1: Expresses x as $x = \lambda\arctan\left(\frac{y}{8}\right)$

dM1: Differentiates to achieve a form $K \times \frac{1}{\left(1+\left(\frac{y}{8}\right)^2\right)}$

ddM1: Differentiates their x **correctly** *using the chain rule* **and** eliminates fractions correctly to achieve the correct form.

A1: cao and depends on all previous marks.

Question Number	Scheme	Marks
5 (a)	Way 1: $\dfrac{\cot^2 x}{1+\cot^2 x} \equiv \dfrac{\cos^2 x \big/ \sin^2 x}{\operatorname{cosec}^2 x} \equiv \dfrac{\cos^2 x \big/ \sin^2 x}{1 \big/ \sin^2 x} \equiv \cos^2 x$	M1, M1, A1
	Way 2: $\dfrac{\cot^2 x}{1+\cot^2 x} \equiv \dfrac{\cos^2 x}{\sin^2 x+\cos^2 x} \equiv \dfrac{\cos^2 x}{1} \equiv \cos^2 x$	M1, M1 A1
	Way 3: Consider $\cos^2 x(1+\cot^2 x) = \cos^2 x \operatorname{cosec}^2 x = \dfrac{\cos^2 x}{\sin^2 x} = \cot^2 x$ Hence $\dfrac{\cot^2 x}{1+\cot^2 x} \equiv \cos^2 x$	M1 M1 A1
	Way 4: Consider $\cos^2 x(1+\cot^2 x) = \cos^2 x(1+\dfrac{\cos^2 x}{\sin^2 x}) = \dfrac{\cos^2 x(\sin^2 x+\cos^2 x)}{\sin^2 x} = \dfrac{\cos^2 x}{\sin^2 x} = \cot^2 x$ Hence $\dfrac{\cot^2 x}{1+\cot^2 x} \equiv \cos^2 x$	M1 M1 A1
		(3)
(b)	$\cos^2 x = 8(2\cos^2 x - 1) + 2\cos x$	M1
	$15\cos^2 x + 2\cos x - 8 = 0$	A1
	So cos$x$ = 2/3 or $-4/5$ leading to at least one value for x	dM1
	$\Rightarrow x = 48.2°$ or $143.1°$ or $216.9°$ or $311.8°$	A1, A1
		(5)
		(8 marks)

Notes

(a)

Way 1 and Way 3:

M1: See one of $1+\cot^2 x = \operatorname{cosec}^2 x$, $\cot^2 x = \cos^2 x \big/ \sin^2 x$ used.

M1: See two of $1+\cot^2 x = \operatorname{cosec}^2 x$, $\cot^2 x = \cos^2 x \big/ \sin^2 x$, $\operatorname{cosec}^2 x = 1 \big/ \sin^2 x$ used.

A1: See all three of $1+\cot^2 x = \operatorname{cosec}^2 x$, $\cot^2 x = \cos^2 x \big/ \sin^2 x$, $\operatorname{cosec}^2 x = 1 \big/ \sin^2 x$ used and **conclusion reached** – which may be reaching $\cos^2 x$ if working from LHS to RHS.

Way 2:

M1: See one of $\sin^2 x(1+\cot^2 x) = \sin^2 x + \cos^2 x$, $\cot^2 x \sin^2 x = \cos^2 x$ used **or implied**

M1: See two of $\sin^2 x(1+\cot^2 x) = \sin^2 x + \cos^2 x$, $\cot^2 x \sin^2 x = \cos^2 x$, $\sin^2 x + \cos^2 x = 1$ used **or implied**

A1: See all three of $\sin^2 x(1+\cot^2 x) = \sin^2 x + \cos^2 x$, $\cot^2 x \sin^2 x = \cos^2 x$, $\sin^2 x + \cos^2 x = 1$ used **or implied** and **conclusion reached**. In this method multiplication of numerator and denominator may be implied and $\sin^2 x(1+\cot^2 x) = \sin^2 x + \cos^2 x$ **may not be explicit**.

Way 4:

M1: See $\cot^2 x = \cos^2 x \big/ \sin^2 x$ used

M1: See both $\cot^2 x = \cos^2 x \big/ \sin^2 x$ and $(1+\dfrac{\cos^2 x}{\sin^2 x}) = \dfrac{(\sin^2 x+\cos^2 x)}{\sin^2 x}$ used

A1: See all three of $\cot^2 x = \cos^2 x \big/ \sin^2 x$ $(1+\dfrac{\cos^2 x}{\sin^2 x}) = \dfrac{(\sin^2 x+\cos^2 x)}{\sin^2 x}$ and $\sin^2 x + \cos^2 x = 1$ used and **conclusion reached**.