Question Number	Scheme		Marks
3.(i)(a)	$\begin{array}{r} x-5 \\ x+2\overline{)\,x^2-3x+6} \\ \underline{x^2+2x} \\ -5x+6 \\ \underline{-5x-10} \\ 16 \end{array}$	$\dfrac{x^2-3x+6}{x+2}=Ax+B+\dfrac{C}{x+2}$ $\Rightarrow x^2-3x+6=(Ax+B)(x+2)+C$ $\Rightarrow 1=A,-3=2A+B,6=2B+C$ $\Rightarrow A=\ldots,\ B=\ldots,\ C=\ldots$	M1
	Quotient = $x-5$ and Remainder = 16		A1
			(2)
(b)	$\displaystyle\int\frac{x^2-3x+6}{x+2}\,\mathrm{d}x=\int x-5+\frac{16}{x+2}\,\mathrm{d}x=\frac{1}{2}x^2-5x+16\ln(x+2)+c$		M1, B1ft, A1
			(3)
(ii)	$\displaystyle\int_0^{\frac{\pi}{6}}(\cos\theta-\sin\theta)^2\mathrm{d}\theta=\int_0^{\frac{\pi}{6}}(\cos^2\theta-2\sin\theta\cos\theta+\sin^2\theta)\mathrm{d}\theta$ **and** one of $\displaystyle\int\sin^2\theta+\cos^2\theta\ \mathrm{d}\theta=\int 1\ \mathrm{d}\theta\to\theta$ or $\displaystyle\int 2\sin\theta\cos\theta\ \mathrm{d}\theta=\int\sin 2\theta\ \mathrm{d}\vartheta\to k\cos 2\theta$ or other valid integration approaches for one of these		M1
	E.g. $\to\displaystyle\int 1-\sin 2\theta\ \mathrm{d}\theta\to\theta+k\cos 2\theta$ (oe)		dM1
	$\displaystyle\int_0^{\frac{\pi}{6}}(\cos\theta-\sin\theta)^2\mathrm{d}\theta=\left[\theta+\frac{1}{2}\cos 2\theta\right]_0^{\frac{\pi}{6}}$ oe		A1
	$\displaystyle=\left(\frac{\pi}{6}+\frac{1}{2}\times\frac{1}{2}\right)-\left(0+\frac{1}{2}\right)=\frac{\pi}{6}-\frac{1}{4}$		ddM1 A1
			(5)
			(10 marks)

Notes

(i)(a)

M1: Full method to find the quotient and remainder. May use long division to produce a linear quotient and a constant remainder or use a partial fraction method to set up correct form and find the constants.

A1: Correct quotient and remainder seen anywhere – even in the division. If using partial fractions with correct form shown allow for the correct constants found.

(b)

M1: $\displaystyle\int\frac{K}{x+2}\,\mathrm{d}x\to A\ln(x+2)$ (A constant) condoning a missing bracket.

B1ft: Correct integration of their linear quotient. Note $(x-5)\to\dfrac{1}{2}(x-5)^2$ is an equivalent correct expression.

A1: Fully correct answer including the constant. Note $\ln|x+2|$ is fine.

(ii)

Note: d θ need not be seen, and condone use of dx in the integrals.

M1: Attempts to multiply out (correct form for terms – at least three terms) and attempts a correct integration method on either the $\cos^2\theta+\sin^2\theta$ or the $\sin\theta\cos\theta$. Must use correct trig identities – but allow benefit of doubt throughout if e.g. $\cos^2\theta$ and $\sin^2\theta$ double angle identities are two correct identities but may have changed order.

There are a variety of methods that can be used:

$\cos^2\theta+\sin^2\theta=1\rightarrow\theta$ is the most common on these terms but also possible is

$\cos^2\theta+\sin^2\theta=\frac{1}{2}(1+\cos2\theta)+\frac{1}{2}(1-\cos2\theta)\rightarrow\frac{\theta}{2}+A\sin2\theta+\frac{\theta}{2}-A\sin2\theta$

For the other term $2\sin\theta\cos\theta=\sin2\theta\rightarrow K\cos2\theta$ is most common, but you may also see $2\sin\theta\cos\theta\rightarrow K\cos^2\theta$ or $K\sin^2\theta$ (or possibly attempts at integration by parts, but they would need to be complete attempts).

dM1: Multiplies out and applies correct integration technique to all terms of the integral.

A1: Fully correct integral, unsimplified. Possible forms are

$\theta+\frac{1}{2}\cos2\theta$, $\frac{\theta}{2}+\frac{1}{4}\sin2\theta+\cos^2\theta+\frac{\theta}{2}-\frac{1}{4}\sin2\theta$, $\theta-\sin^2\theta$ etc.

Condone a mix of x's and θ's as long as they are recovered by substitution of the limits, but A0 if they do not attempt the substitution.

ddM1: Applies the limits to their full attempt at the integral and evaluates trig expressions– depends on both previous Ms. Both limits must be applied, do not accept just "– 0" for the lower limit if it is not 0

A1: $\frac{\pi}{6}-\frac{1}{4}$ oe simplified expression, e.g $\frac{2\pi-3}{12}$

Question Number	Scheme		Marks
4	Differentiates wrt x $\quad \frac{dy}{dx} = 16\sec^2(2x)$	Gives $x = \frac{1}{2}\arctan\left(\frac{y}{8}\right)$	M1
	Inverts to get $\frac{dx}{dy} = \frac{1}{16\sec^2 2x} = \frac{1}{16(1+\tan^2 2x)}$ Or $\frac{dy}{dx} = 16\left(1+\tan^2 2x\right) = 16\times\left(1+\left(\frac{y}{8}\right)^2\right)$	$\frac{dx}{dy} = K \times \frac{1}{\left(1+\left(\frac{y}{8}\right)^2\right)}$	dM1
	$\frac{dx}{dy} = \frac{A}{B+y^2}$	$\frac{dx}{dy} = \frac{1}{2}\cdot\frac{1/8}{(1+\left(\frac{y}{8}\right)^2)} = \frac{A}{B+y^2}$	ddM1
	$= \frac{4}{64+y^2}$		A1
			(4)
			(4 marks)

Notes

Way 1:

M1: Achieves $\left(\frac{dy}{dx}\right) = \lambda\sec^2(2x)$ or $1 = \lambda\sec^2(2x)\frac{dx}{dy}$ (oe) In the second case the $\frac{dx}{dy}$ must be present, but for this mark allow if just "$=\lambda\sec^2 2x$" is given – but the $\frac{dy}{dx}$ must be present for further marks to be scored.

If they change tan 2x to $\frac{\sin 2x}{\cos 2x}$ they can score this mark for $\frac{dy}{dx} = \frac{a\cos 2x\cos 2x \pm b\sin 2x\sin 2x}{(\cos 2x)^2}$

If other identities are seen progress is unlikely and must reach a derivative in terms of trig terms in 2x.

For instance, use of $y = \frac{16\tan x}{1-\tan^2 x}$ will not like make suitable porgress as they will be working in tan$x$. Only if suitable identities to get in to 2x terms is made will they become elligible for this mark. Review any cases you feel worthy of merit and do not know how to score.

dM1: Either

- finds reciprocal of their function (or makes $\frac{dx}{dy}$ the subject) **and** uses $1+\tan^2 2x = \sec^2 2x$

OR

- uses $1+\tan^2 2x = \sec^2 2x$ **and** replaces tan 2x by y/8 to get and expression in y only

ddM1: Full process (all three steps noted in the dM mark) to make $\frac{dx}{dy}$ the subject in terms of y and eliminate fractions to reach the correct form. Note, correct replacement of tan2x by y/8 is required but there may be algebraic slips following that in simplifying to the correct form.

A1: cao and depends on all previous marks.

Way 2:

M1: Expresses x as $x = \lambda\arctan\left(\frac{y}{8}\right)$

dM1: Differentiates to achieve a form $K \times \frac{1}{\left(1+\left(\frac{y}{8}\right)^2\right)}$

ddM1: Differentiates their x **correctly** *using the chain rule* **and** eliminates fractions correctly to achieve the correct form.

A1: cao and depends on all previous marks.