Question Number	Scheme	Marks
8	$x = \mathrm{e}^{2\tan y} \Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}y} = \mathrm{e}^{2\tan y} \times 2\sec^2 y$	B1
	$\Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}y} = 2x\left(1+\tan^2 y\right) = 2x\left(1+\left(\dfrac{\ln x}{2}\right)^2\right)$	M1
	$\Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{4}{2x\left(4+\left(\ln x\right)^2\right)}$	M1
	$\Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{2}{x\left(4+\left(\ln x\right)^2\right)}$	A1
		(4) **(4 marks)**

Note: Condone incorrect notations for $\left(\ln x\right)^2$ such as $\ln x^2$ and $\ln^2 x$ in working, but not in the final answer.

B1 Correct expression for $\dfrac{\mathrm{d}x}{\mathrm{d}y}$ as in scheme or in an equivalent form

The following two M marks require their $\dfrac{\mathrm{d}x}{\mathrm{d}y}$ to be in the form $k\,\mathrm{e}^{2\tan y}\sec^2 y$.

M1 Either:

- Attempts the reciprocal of their $\dfrac{\mathrm{d}x}{\mathrm{d}y}$ to form an expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$; OR
- Uses $\sec^2 y = 1+\tan^2 y$ to get $\dfrac{\mathrm{d}x}{\mathrm{d}y}$ in terms of x.

M1 Both:

- Attempts the reciprocal of their $\dfrac{\mathrm{d}x}{\mathrm{d}y}$ to form an expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$; AND
- Uses $\sec^2 y = 1+\tan^2 y$ to get $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x.

A1 $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{2}{x\left(4+\left(\ln x\right)^2\right)}$. Condone missing $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ on LHS as long as it appears somewhere in working and it is clear that it refers to their final answer.

Alt for the two M marks using cos

The following two M marks require their $\dfrac{\mathrm{d}x}{\mathrm{d}y}$ to be in the form $\dfrac{k\,\mathrm{e}^{2\tan y}}{\cos^2 y}$.

M1 Either:

- Attempts the reciprocal of their $\dfrac{\mathrm{d}x}{\mathrm{d}y}$ to form an expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$; OR
- Forms an expression for $\cos^2 y$ using a valid method and gets $\dfrac{\mathrm{d}x}{\mathrm{d}y}$ in terms of x.

M1 Both:

- Attempts the reciprocal of their $\frac{\mathrm{d}x}{\mathrm{d}y}$ to form an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$; AND
- Forms an expression for $\cos^2 y$ using a valid method and gets $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x.

Alt using arctan. Note that B1 M1 M0 A0 is not possible via this route

B1 $x = \mathrm{e}^{2\tan y} \Rightarrow y = \arctan\left(\frac{1}{2}\ln x\right) \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{1+\left(\frac{1}{2}\ln x\right)^2} \times \ldots$ where … could be 1.

M2 $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{1+\left(\frac{1}{2}\ln x\right)^2} \times \frac{1}{2x}$

A1 $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2}{x\left(4+\left(\ln x\right)^2\right)}$

Alt using implicit differentiation

B1 $x = \mathrm{e}^{2\tan y} \Rightarrow \ln x = 2\tan y \Rightarrow \frac{1}{x}\frac{\mathrm{d}x}{\mathrm{d}y} = 2\sec^2 y$ OR $x = \mathrm{e}^{2\tan y} \Rightarrow \ln x = 2\tan y \Rightarrow \frac{1}{x} = 2\sec^2 y \frac{\mathrm{d}y}{\mathrm{d}x}$

Other variations are possible. Then as main method

Question Number	Scheme	Marks
9 (a)	$\mathrm{h}(x)=8+3\sin x(\cos x-2\sin x)$	
	$\mathrm{h}(x)=8+3\sin x\cos x-6\sin^2 x$	
	$\mathrm{h}(x)=8+\frac{3}{2}\sin 2x+3\cos 2x-3$ o.e.	M1, dM1
	$R\cos\alpha=\frac{3}{2},\ R\sin\alpha=3$	A1
	Let $\frac{3}{2}\sin 2x+3\cos 2x=R\sin(2x+\alpha)$	
	$R^2=\left("\frac{3}{2}"\right)^2+"3"^2\Rightarrow R=\ldots$	ddM1
	$\tan\alpha=\frac{"3"}{\left("\frac{3}{2}"\right)}\Rightarrow\alpha=\ldots$	ddM1
	$\left(\mathrm{h}(x)=\right)5+\frac{3}{2}\sqrt{5}\sin(2x+1.107)$	A1
		(6)
(b)	$5-\frac{3}{2}\sqrt{5}\leqslant \mathrm{h}(x)\leqslant 5+\frac{3}{2}\sqrt{5}$	B1ft
		(1)
(c)	$2x+"1.107"=\frac{7\pi}{2}\Rightarrow x=\ldots$	M1
	$x=$ awrt 4.94	A1
		(2)
		(9 marks)

(a)

M1 Multiplies out bracket and attempts to use **either** of

- $\sin 2x=2\sin x\cos x$
- $\cos 2x=1-2\sin^2 x$ o.e.

dM1 Dependent on the first M1. Multiplies out bracket and attempts to use **both** of

- $\sin 2x=2\sin x\cos x$
- $\cos 2x=1-2\sin^2 x$ o.e.

Expands $R\sin(2x+\alpha)$ and compares coefficients to reach two equations in R and α. In some cases, the two equations may not be explicitly stated, but implied by further work.

Alt for the first two marks using the given form of the answer, expanding and equating coefficients.

M1 Expands $R\sin(2x+\alpha)$ to $R\sin 2x\cos\alpha+R\cos 2x\sin\alpha$ and uses **either** $\sin 2x=2\sin x\cos x$ or $\cos 2x=1-2\sin^2 x$

dM1 Dependent on the first M1. Expands using **both** $\sin 2x=2\sin x\cos x$ and $\cos 2x=1-2\sin^2 x$ to reach $2R\sin x\cos x\cos\alpha+R\sin\alpha-2R\sin^2 x\sin\alpha$

And compares coefficients of $\sin x\cos x$ and $\sin^2 x$ to reach two equations in R and α. In some cases, the two equations may not be explicitly stated, but implied by further work.