Question Number	Scheme	Marks
9 (a)	$\mathrm{h}(x)=8+3\sin x(\cos x-2\sin x)$	
	$\mathrm{h}(x)=8+3\sin x\cos x-6\sin^2 x$	
	$\mathrm{h}(x)=8+\frac{3}{2}\sin 2x+3\cos 2x-3$ o.e.	M1, dM1
	$R\cos\alpha=\frac{3}{2},\ R\sin\alpha=3$	A1
	Let $\frac{3}{2}\sin 2x+3\cos 2x=R\sin(2x+\alpha)$	
	$R^2=\left("\frac{3}{2}"\right)^2+"3"^2\Rightarrow R=\ldots$	ddM1
	$\tan\alpha=\frac{"3"}{\left("\frac{3}{2}"\right)}\Rightarrow\alpha=\ldots$	ddM1
	$\left(\mathrm{h}(x)=\right)5+\frac{3}{2}\sqrt{5}\sin(2x+1.107)$	A1
		(6)
(b)	$5-\frac{3}{2}\sqrt{5}\leqslant \mathrm{h}(x)\leqslant 5+\frac{3}{2}\sqrt{5}$	B1ft
		(1)
(c)	$2x+"1.107"=\frac{7\pi}{2}\Rightarrow x=\ldots$	M1
	$x=$ awrt 4.94	A1
		(2)
		(9 marks)

(a)

M1 Multiplies out bracket and attempts to use **either** of

- $\sin 2x=2\sin x\cos x$
- $\cos 2x=1-2\sin^2 x$ o.e.

dM1 Dependent on the first M1. Multiplies out bracket and attempts to use **both** of

- $\sin 2x=2\sin x\cos x$
- $\cos 2x=1-2\sin^2 x$ o.e.

Expands $R\sin(2x+\alpha)$ and compares coefficients to reach two equations in R and α. In some cases, the two equations may not be explicitly stated, but implied by further work.

Alt for the first two marks using the given form of the answer, expanding and equating coefficients.

M1 Expands $R\sin(2x+\alpha)$ to $R\sin 2x\cos\alpha+R\cos 2x\sin\alpha$ and uses **either** $\sin 2x=2\sin x\cos x$ or $\cos 2x=1-2\sin^2 x$

dM1 Dependent on the first M1. Expands using **both** $\sin 2x=2\sin x\cos x$ and $\cos 2x=1-2\sin^2 x$ to reach $2R\sin x\cos x\cos\alpha+R\sin\alpha-2R\sin^2 x\sin\alpha$

And compares coefficients of $\sin x\cos x$ and $\sin^2 x$ to reach two equations in R and α. In some cases, the two equations may not be explicitly stated, but implied by further work.

A1 $R\cos\alpha = \frac{3}{2}$ and $R\sin\alpha = 3$ or equivalent equations. Note that a value of R may be embedded if it has already been calculated by inspection of coefficients or construction of a triangle. In some cases, the two equations may not be explicitly stated, but implied by further work.

If no equations are given and subsequent calculations of R and α are correct, give the first A mark and potentially the final A mark (the correct equations are implied).

Special Case. If equations are given in which the only error is the missing R (e.g. $\cos\alpha = \frac{3}{2}$ and $\sin\alpha = 3$) then allow the first two M marks, but give the first A0. The ddM marks below are available for an otherwise correct calculation of R and α; and the final A mark is also available for an otherwise fully correct solution. This SC could also be applied if an incorrect variable is used, e.g. x instead of α.

The following two M marks are both dependent on the first two M marks, i.e. we require a correct method leading to $R\cos\alpha = "\frac{3}{2}"$ and $R\sin\alpha = "3"$.

ddM1 $R^2 = \left("\frac{3}{2}"\right)^2 + "3"^2 \Rightarrow R = ...$ May be implied by their R value, even if R is inexact.

ddM1 $\tan\alpha = \frac{"3"}{\left("\frac{3}{2}"\right)} \Rightarrow \alpha = ...$ but condone $\tan\alpha = \frac{\left("\frac{3}{2}"\right)}{"3"} \Rightarrow \alpha = ...$ Alternatively α can be found using $\sin\alpha = \frac{"3"}{R}$ or $\cos\alpha = \frac{"3/2"}{R}$ with their R.

A1 $\left(\mathrm{h}(x) =\right) 5 + \frac{3}{2}\sqrt{5}\sin\left(2x + 1.107\right)$. A full expression must be given for h(x), not just values of P, R and α. This expression must be seen in part (a).

(b) **Note: this mark can be scored following incorrect working in part (a).**

B1ft $5 - \frac{3}{2}\sqrt{5} \leqslant \mathrm{h}(x) \leqslant 5 + \frac{3}{2}\sqrt{5}$ o.e. allowing follow through on their P and R

$\left[5 - \frac{3\sqrt{5}}{2}, 5 + \frac{3\sqrt{5}}{2}\right]$ and $\left\{\mathrm{h}(x) : 5 - \frac{3\sqrt{5}}{2} \leqslant \mathrm{h}(x) \leqslant 5 + \frac{3\sqrt{5}}{2}\right\}$ are suitable alternatives.

Condone y or h in place of h(x) but not f(x) or x.

(c) **Note: full marks can be scored in part (c) following incorrect working in part (a).**

M1 Attempts to solve $2x + "1.107" = \frac{7\pi}{2}$ using the correct order of operations, leading to a value for x.

Alternatively solves $\sin\left(2x + "1.107"\right) = -1$ using the correct order of operations, and selects the second positive value of x.

Or differentiates to reach $\cos\left(2x + "1.107"\right) = 0$ and solves to find the fourth positive solution.

A1 $x =$ awrt 4.94

Question Number	Scheme	Marks
10 (a)	$\left(\frac{\mathrm{d}N_A}{\mathrm{d}t}\right) = 900\times 0.2\mathrm{e}^{0.2\times 5} = \text{awrt } 489 \text{ or } 180\mathrm{e}$	M1, A1 **(2)**
(b)	States or implies that $P = 2\,000$	B1
	$11570 = 8000 + 2000\mathrm{e}^{k\times 4} \Rightarrow 2000\mathrm{e}^{4k} = 3570$	M1
	$\Rightarrow k = \text{awrt } 0.145$	dM1, A1 **(4)**
(c)	$8000 + 900\mathrm{e}^{0.2T} = 8000 + "2000"\mathrm{e}^{"0.145"T}$	
	$\mathrm{e}^{(0.2-"0.145")T} = \frac{"2000"}{900}$ or $\ln\left(900\mathrm{e}^{0.2T}\right) = \ln\left("2000"\mathrm{e}^{"0.145"T}\right)$	M1
	$\ln 900 + 0.2T = \ln"2000" + "0.145"T$	
	$T = \frac{1}{(0.2-"0.145")}\ln\left(\frac{"2000"}{900}\right)$	dM1
	$T = 14.5$	A1 **(3)** **(9 marks)**

(a) **Note: if a correct answer is seen without any working, please send to review.**

M1 $\frac{\mathrm{d}N_A}{\mathrm{d}t} = C\mathrm{e}^{0.2\times 5} = \ldots$ where $C \neq 900$. Withhold this mark if they substitute $t = 5$ before differentiation.

A1 awrt 489 or exact 180e. isw

(b)

B1 States or implies that $P = 2\,000$

M1 Proceeds to $A\mathrm{e}^{4k} = B$ where $A \times B > 0$

dM1 Dependent on previous M1. Correct attempt to solve using ln, following the correct order of operations. Condone division the wrong way round, i.e. $A\mathrm{e}^{4k} = B \Rightarrow \mathrm{e}^{4k} = \frac{A}{B}$. Proceeding from $A\mathrm{e}^{4k} = B$ to a value for *k* with no ln work is dM0 A0.

A1 $k = \text{awrt } 0.145$. Exact answer only is A0.

(c) **Condone use of *t* in place of *T* throughout this part.**

M1 Cancels 8000 on each side, then **either**:

- uses correct index work to achieve an equation in $\mathrm{e}^{(0.2-"0.145")T}$ that could be rearranged to the form $\mathrm{e}^{(0.2-"0.145")T} = K$ where $K > 0$; **or**
- takes ln of both sides and uses the log addition law on both sides, giving an equation involving two terms in *T* and at least one constant term.

dM1 Dependent on previous M1. Correct ln work leading to a positive value or positive exact expression for *T*. Proceeding from $\mathrm{e}^{(0.2-"0.145")T} = K$ to a value for *T* with no ln work is dM0 A0.

A1 *T* = awrt 14.5. Exact answer only is A0.