(b)	$2\cot x = 2\sqrt{3} \Rightarrow \tan x = \dfrac{1}{\sqrt{3}}$ $x = \arctan\left(\dfrac{1}{\sqrt{3}}\right) \Rightarrow x = \dfrac{\pi}{6}$	M1A1
	$y = \ln\left(1-\cos\left(\dfrac{2\pi}{6}\right)\right) = \ln\dfrac{1}{2}$ or $-\ln 2$	M1A1
		(8 marks)

Notes

If $k = 2$ is obtained fortuitously or guessed in part (a) do not allow the A marks in part (b).

M1: Uses $\cot x = \dfrac{1}{\tan x}$ to find $\tan x$ and then arctan to find x which may be implied by their value of x. Note that some candidates may not be able to find a value of x depending on their value of k from part (a) so allow this mark for e.g. $k\cot x = 2\sqrt{3} \Rightarrow \tan x = \dfrac{k}{2\sqrt{3}} \Rightarrow x = \tan^{-1}\left(\dfrac{k}{2\sqrt{3}}\right)$ with their numerical k.

A1: $x = \dfrac{\pi}{6}$ following $k = 2$ obtained correctly in part (a). (Do not accept $30°$ for this mark)

Must be evaluated e.g. not $x = \tan^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$.

Ignore additional (incorrect) values such as $x = \dfrac{5\pi}{6}$

M1: Substitutes their value of x in $y = \ln(1-\cos 2x)$ to find the y coordinate.

May be implied by their value and allow e.g. $y = \ln\left(1-\cos 2\left(\tan^{-1}\left(\dfrac{k}{2\sqrt{3}}\right)\right)\right)$ with their numerical k.

A1: $y = \ln\dfrac{1}{2}$ or $-\ln 2$ following $x = \dfrac{\pi}{6}$ or $30°$ from fully correct work including $k = 2$ obtained correctly in part (a).

Must be evaluated e.g. not $y = \ln\left(1-\cos 2\left(\tan^{-1}\left(\dfrac{1}{\sqrt{3}}\right)\right)\right)$.

Withhold this mark if other values of x and y are given within the range.

(4)

Question Number	Scheme	Marks
5(a)	$2\text{cosec}2A = \dfrac{2}{\sin 2A}$	B1
	$\dfrac{2}{\sin 2A} - \cot A = \dfrac{2}{2\sin A\cos A} - \dfrac{\cos A}{\sin A}$	M1
	$= \dfrac{2-2\cos^2 A}{2\sin A\cos A}$	M1
	$= \dfrac{2\sin^2 A}{2\sin A\cos A} = \dfrac{\sin A}{\cos A} = \tan A$ *	A1*
		(4)

Notes

Condone use of a different variable e.g. θ or x for A.

Condone the use of mixed variables for the B and M marks as long as the intention is clear.

B1: Writes $\text{cosec}2A$ as $\dfrac{1}{\sin 2A}$

M1: Uses $\sin 2A = 2\sin A\cos A$ or e.g. $\sin 2A = \sin A\cos A + \cos A\sin A$ and $\cot A = \dfrac{\cos A}{\sin A}$ to write the lhs in terms of sin A and cos A only.

M1: For writing the lhs as a single fraction in terms of sin A and cos A only.

It is for writing $\dfrac{a}{b\sin A\cos A} - \dfrac{\cos A}{\sin A}$ as a single fraction with a correct common denominator (which may not be the lowest) and 2 terms in the numerator, one of which must have been "modified".

e.g. $\dfrac{2}{2\sin A\cos A} - \dfrac{\cos A}{\sin A} = \dfrac{2\sin A - 2\sin A\cos^2 A}{2\sin^2 A\cos A}$

is acceptable without a lowest common denominator

e.g. $\dfrac{2}{2\sin A\cos A} - \dfrac{\cos A}{\sin A} = \dfrac{\sin A - 2\cos^2 A}{2\sin A\cos A}$

is acceptable with a correct denominator and 2 terms in the numerator of which at least 1 has been modified

A1*: Completes the proof with no errors.

The 2's must be cancelled at some point but it is ok for them to just "disappear".

The $1-\cos^2 A$ in the numerator must be replaced with $\sin^2 A$ and the expression $\dfrac{\sin A}{\cos A}$ must be seen before being replaced with tan A but allow this appear in the form of cancelling e.g. $\dfrac{\cancel{2}\sin^{\cancel{2}} A}{\cancel{2}\cancel{\sin A}\cos A}$. For this mark condone minor notational errors (e.g. a missing A) but not consistent poor notation throughout such as mixed variables or writing $\cos^2 A$ as $\cos A^2$.

For the final 2 marks, allow a "meet in the middle" approach

e.g. sets $\dfrac{2}{2\sin A\cos A} - \dfrac{\cos A}{\sin A} = \dfrac{\sin A}{\cos A}$

$\times 2\sin A\cos A \rightarrow 2 - 2\cos^2 A = 2\sin^2 A \Rightarrow 2\sin^2 A = 2\sin^2 A$ Hence proven.

Score the first B1M1 as in the main scheme and then M1 for e.g. multiplying through by $2\sin A\cos A$ and A1* for a fully correct proof with a (minimal) conclusion.

(b)(i)	$2\text{cosec}4\theta - \cot 2\theta = \sqrt{3} \Rightarrow \tan 2\theta = \sqrt{3} \Rightarrow \theta = ...$	M1
	$\Rightarrow \theta = \dfrac{\arctan\sqrt{3}}{2} = \dfrac{\pi}{6}$ or awrt 0.524	A1

M1: Uses part (a) to write the given equation in form $\tan 2\theta = \sqrt{3}$ and proceeds to a value for θ by finding arctan of $\sqrt{3}$ and dividing by 2.

It may be implied by e.g. $A = 2\theta \rightarrow \tan A = \sqrt{3} \Rightarrow A = ...$ as long as they then find a value for θ by finding arctan of $\sqrt{3}$ and dividing by 2.

If a candidate restarts then they must reach $\tan 2\theta = \sqrt{3}$

A1: Achieves $\theta = \dfrac{\pi}{6}$ or accept awrt 0.524 but not 30°

Ignore extra solutions outside the range but withhold this mark if there are extra solutions in range. Must follow the award of M1 so correct answer only scores no marks.

(ii)	$\tan\theta + \cot\theta = 5 \Rightarrow \sin 2\theta = \dfrac{2}{5}$	M1
	$\Rightarrow \theta = \dfrac{1}{2}\arcsin\left(\dfrac{2}{5}\right) =$ awrt 0.206,1.37	M1 A1
		(5)
		(9 marks)

M1: Uses part (a) to deduce that $\sin 2\theta = \dfrac{2}{5}$

M1: Uses the correct order of operations to find at least one angle for θ

It is for $\sin 2\theta = k$ where $|k| < 1$ followed by $\theta = \dfrac{1}{2}\arcsin(k) = ...$

If working is not shown you may need to check.

A1: For awrt 0.206 and awrt 1.37 and no other values in the range.

Accept awrt 0.0656π and awrt 0.436π.

Apply isw if values are subsequently rounded further.

Answers in degrees score A0 and correct answers with no working scores no marks.

Special cases:

$$\tan\theta + \cot\theta = 5 \Rightarrow \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = 5 \Rightarrow \frac{1}{\frac{1}{2}\sin 2\theta} = 5 \Rightarrow \sin 2\theta = \frac{2}{5}$$

Scores M1 for reaching $\sin 2\theta = \dfrac{2}{5}$ as in the main scheme and then **d**M1A1 as above.

BUT

$$\tan\theta + \cot\theta = 5 \Rightarrow \tan\theta + \frac{1}{\tan\theta} = 5 \Rightarrow \tan^2\theta - 5\tan\theta + 1 = 0$$

$$\Rightarrow \tan\theta = \frac{5 \pm \sqrt{21}}{2} \Rightarrow \theta = \text{awrt } 0.206, 1.37$$

Scores M1 for reaching $\tan\theta = \dfrac{5 \pm \sqrt{21}}{2}$ and obtains awrt 0.206 **or** awrt 1.37

and then M1 for reaching $\tan\theta = \dfrac{5 \pm \sqrt{21}}{2}$ and obtains awrt 0.206 **and** awrt 1.37

The A1 is **not** available as they have not used part (a).