(b)	$g'(x) = 2x - \dfrac{3}{(x-3)^2}$ **or** $g'(x) = \dfrac{\left(x^2+x-12\right)\left(4x^3+3x^2-14x+8\right)-\left(x^4+x^3-7x^2+8x-48\right)(2x+1)}{\left(x^2+x-12\right)^2}$	M1A1ft
	$x=4 \Rightarrow g'(4) = 2\times 4 - \dfrac{3}{(4-3)^2} = 5$	M1
	$x=4 \Rightarrow g(4) = 4^2+5+\dfrac{3}{4-3}(=24)$ $\Rightarrow y-24=5(x-4)$ **or e.g.** $y=5x+c \Rightarrow 24=5\times 4+c \Rightarrow c=...$	**d**M1
	$y=5x+4$	A1
		(5)
		(9 marks)

Notes

Note that candidates should not be using a calculator in this part so if there is no algebraic differentiation, they will generally score no marks.

M1: Differentiates their $x^2 + A + \dfrac{B}{x-3}$ to obtain $2x \pm \dfrac{B}{(x-3)^2}$ oe e.g. $2x \pm B(x-3)^{-2}$

or differentiates $\dfrac{x^4+x^3-7x^2+8x-48}{x^2+x-12}$ to obtain an expression of the form

$$g'(x) = \frac{\left(x^2+x-12\right)\times \text{a cubic polynomial} - \left(x^4+x^3-7x^2+8x-48\right)\times \text{a 2 term linear expression}}{\left(x^2+x-12\right)^2}$$

A1ft: $x^2 + A + \dfrac{B}{x-3} \rightarrow 2x - \dfrac{B}{(x-3)^2}$. Follow through on their B or allow the letter B or a made up B.

or $g'(x) = \dfrac{\left(x^2+x-12\right)\left(4x^3+3x^2-14x+8\right)-\left(x^4+x^3-7x^2+8x-48\right)(2x+1)}{\left(x^2+x-12\right)^2}$

which must be correct (nothing to follow through).

M1: Subtitutes $x=4$ into their $g'(x)$ in an attempt to find a numerical gradient.
May be implied by their $g'(4)$ following an attempt to differentiate.

dM1: For the correct method of finding an equation of the tangent with

- $x = 4$
- y an attempt at g(4) which may be implied by their value or by 24
- the gradient an attempt at $g'(4)$ following an attempt to differentiate
- the values correctly placed

If $y = mx + c$ is used they must proceed to a value for c.
Depends on the previous method mark.

A1: $y = 5x + 4$ cao and **cso**
Note that an incorrect value for A in part (a) will give the correct answer in (b) but this mark is cso and should be withheld in such cases.

Question Number	Scheme	Marks
4(a)	$y=\ln(1-\cos 2x)\Rightarrow\left(\frac{dy}{dx}=\right)\frac{2\sin 2x}{1-\cos 2x}$	M1A1
	$\Rightarrow\left(\frac{dy}{dx}=\right)\frac{4\sin x\cos x}{1-\left(1-2\sin^2 x\right)}$	M1
	$=\frac{4\sin x\cos x}{2\sin^2 x}=2\cot x$	A1
		(4)

Notes

Note that "$\frac{dy}{dx}=$" is not required apart from in alternative 3 below.

M1: For differentiating $\ln(1-\cos 2x)$ to obtain $\frac{\pm A\sin 2x}{1-\cos 2x}$

A1: $\left(\frac{dy}{dx}=\right)\frac{2\sin 2x}{1-\cos 2x}$ oe

M1: Uses the correct double angle identities $\sin 2x=2\sin x\cos x$ and $\cos 2x=1-2\sin^2 x$
The identity for $\cos 2x$ may be implied by poor bracketing in the denominator e.g.
$1-1-2\sin^2 x$

A1: Simplifies to obtain $\left(\frac{dy}{dx}=\right)2\cot x$ following M1A1M1 previously.

There must be at least one intermediate correct line between $\frac{4\sin x\cos x}{1-\left(1-2\sin^2 x\right)}$ and $2\cot x$

which will usually be $\frac{4\sin x\cos x}{2\sin^2 x}$ or e.g. $\frac{2\cos x}{\sin x}$

	(a) Alternative 1	
	$y=\ln(1-\cos 2x)=\ln\left(2\sin^2 x\right)\Rightarrow\left(\frac{dy}{dx}=\right)\frac{4\sin x\cos x}{2\sin^2 x}$	M1A1
	$\frac{4\sin x\cos x}{2\sin^2 x}=2\cot x$	M1A1

Notes

M1: Writes $\ln(1-\cos 2x)$ as $\ln(2\sin^2 x)$ and differentiates to obtain $\frac{\pm A\sin x\cos x}{\sin^2 x}$

A1: $\left(\frac{dy}{dx}=\right)\frac{4\sin x\cos x}{2\sin^2 x}$ oe

M1: Obtains $\left(\frac{dy}{dx}=\right)\frac{\pm A\sin x\cos x}{\sin^2 x}$ and simplifies to obtain $k\cot x$

A1: Obtains $\left(\frac{dy}{dx}=\right)2\cot x$ following M1A1M1 previously.

	(a) Alternative 2	
	$y=\ln(1-\cos 2x)=\ln\left(2\sin^2 x\right)=\ln 2+2\ln\sin x \Rightarrow \left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)0+\frac{2\cos x}{\sin x}$	M1A1
	$\frac{2\cos x}{\sin x}=2\cot x$	M1A1

Notes

M1: Writes $\ln(1-\cos 2x)$ as $\ln(2\sin^2 x)$ then $\ln 2+2\ln\sin x$ and differentiates to obtain $\frac{\pm A\cos x}{\sin x}$

A1: $\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)\frac{2\cos x}{\sin x}$ oe

M1: Obtains $\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)\frac{\pm A\cos x}{\sin x}$ and simplifies to obtain $k\cot x$

A1: Obtains $\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)2\cot x$ following M1A1M1 previously.

	(a) Alternative 3	
	$y=\ln(1-\cos 2x)\Rightarrow \mathrm{e}^y=1-\cos 2x \Rightarrow \mathrm{e}^y\frac{\mathrm{d}y}{\mathrm{d}x}=2\sin 2x$	M1A1
	$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2\sin 2x}{\mathrm{e}^y}=\frac{2\sin 2x}{1-\cos 2x}=\frac{4\sin x\cos x}{1-\left(1-2\sin^2 x\right)}$	M1
	$\frac{4\sin x\cos x}{2\sin^2 x}=2\cot x$	A1

Notes

M1: Writes $y=\ln(1-\cos 2x)$ as $\mathrm{e}^y=1-\cos 2x$ and differentiates to obtain $\mathrm{e}^y\frac{\mathrm{d}y}{\mathrm{d}x}=A\sin 2x$

A1: $\mathrm{e}^y\frac{\mathrm{d}y}{\mathrm{d}x}=2\sin 2x$ oe

M1: Divides by $\mathrm{e}^y=1-\cos 2x$ to obtain $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\pm A\sin 2x}{1-\cos 2x}$ and uses the correct double angle identities $\sin 2x=2\sin x\cos x$ and $\cos 2x=1-2\sin^2 x$

The identity for $\cos 2x$ may be implied by poor bracketing in the denominator e.g. $1-1-2\sin^2 x$

A1: Simplifies to obtain $\frac{\mathrm{d}y}{\mathrm{d}x}=2\cot x$ following M1A1M1 previously.

There must be at least one intermediate correct line between $\frac{4\sin x\cos x}{1-\left(1-2\sin^2 x\right)}$ and $2\cot x$

which will usually be $\frac{4\sin x\cos x}{2\sin^2 x}$ or e.g. $\frac{2\cos x}{\sin x}$

(b)	$2\cot x = 2\sqrt{3} \Rightarrow \tan x = \dfrac{1}{\sqrt{3}}$ $x = \arctan\left(\dfrac{1}{\sqrt{3}}\right) \Rightarrow x = \dfrac{\pi}{6}$	M1A1
	$y = \ln\left(1-\cos\left(\dfrac{2\pi}{6}\right)\right) = \ln\dfrac{1}{2}$ or $-\ln 2$	M1A1
		(8 marks)

Notes

If $k = 2$ is obtained fortuitously or guessed in part (a) do not allow the A marks in part (b).

M1: Uses $\cot x = \dfrac{1}{\tan x}$ to find $\tan x$ and then arctan to find x which may be implied by their value of x. Note that some candidates may not be able to find a value of x depending on their value of k from part (a) so allow this mark for e.g. $k\cot x = 2\sqrt{3} \Rightarrow \tan x = \dfrac{k}{2\sqrt{3}} \Rightarrow x = \tan^{-1}\left(\dfrac{k}{2\sqrt{3}}\right)$ with their numerical k.

A1: $x = \dfrac{\pi}{6}$ following $k = 2$ obtained correctly in part (a). (Do not accept $30°$ for this mark)

Must be evaluated e.g. not $x = \tan^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$.

Ignore additional (incorrect) values such as $x = \dfrac{5\pi}{6}$

M1: Substitutes their value of x in $y = \ln(1-\cos 2x)$ to find the y coordinate.

May be implied by their value and allow e.g. $y = \ln\left(1-\cos 2\left(\tan^{-1}\left(\dfrac{k}{2\sqrt{3}}\right)\right)\right)$ with their numerical k.

A1: $y = \ln\dfrac{1}{2}$ or $-\ln 2$ following $x = \dfrac{\pi}{6}$ or $30°$ from fully correct work including $k = 2$ obtained correctly in part (a).

Must be evaluated e.g. not $y = \ln\left(1-\cos 2\left(\tan^{-1}\left(\dfrac{1}{\sqrt{3}}\right)\right)\right)$.

Withhold this mark if other values of x and y are given within the range.

(4)