(c)	Chooses interval e.g. $[0.72825, 0.72835]$ AND function, e.g. $f(x) = 2x^2 - 3 + 4x\sin x$ and attempts the value of $f(0.72825)$ and $f(0.72835)$	M1
	$2\times0.72825^2 - 3 + 4\times0.72825\sin 0.72825 = -0.0005$ $2\times0.72835^2 - 3 + 4\times0.72835\sin 0.72835 = 0.00026$ + reason + conclusion	A1
		(2)
		(10 marks)

Notes

(c)

M1: Chooses

- A suitable interval e.g. $[0.72825, 0.72835]$
- A suitable function and attempts to find the value of the function at both ends.

The function may be implied by embedded values e.g.

$2(0.72825)^2 - 3 + 4(0.72825)\sin(0.72825)$ and $2(0.72835)^2 - 3 + 4(0.72835)\sin(0.72835)$

or by their working/values e.g. $0.72825 \rightarrow \frac{dy}{dx} = \ldots$, $0.72835 \rightarrow \frac{dy}{dx} = \ldots$,

Accept as suitable functions, $\pm(2\alpha^2 - 3 + 4\alpha\sin\alpha)$, $\pm\left(x - \frac{3}{(2x + 4\sin x)}\right)$, their $\frac{dy}{dx}$

Allow a smaller interval containing the root.

The interval does not have to be stated so the values may be seen at the point of substitution.

Condone the use of degree mode – see table below for values.

A1: Achieves

- Correct values to 1 sf rounded or truncated
- Gives a valid reason e.g. change of sign (or equivalent e.g. > 0, < 0)
- Minimal conclusion such as "hence root", "turning point", "proven"

There is no requirement to reference continuity and any mention of it can be ignored.

For reference:

Function f	f (0.72825)	f (0.72835)
$2x^2 - 3 + 4x\sin x$	–0.00051431… (Degrees = –1.90227…)	0.00026066… (Degrees = –1.90197…)
$x - \frac{3}{(2x + 4\sin x)}$	–0.00012487… (Degrees = –1.26201…)	0.000063278… (Degrees = –1.26163…)
$\frac{dy}{dx}$	–0.0002945… (Degrees = –0.95117…)	0.000149267 (Degrees = –0.95102…)

Attempts at repeated iteration score no marks in (c).

Question Number	Scheme	Marks
3(a)	$$\begin{array}{r} x^2+5 \\ x^2+x-12\overline{)\,x^4+x^3-7x^2+8x-48} \\ \underline{x^4+x^3-12x^2} \\ 5x^2+8x-48 \\ \underline{5x^2+5x-60} \\ 3x+12 \end{array}$$	M1A1
	$\dfrac{x^4+x^3-7x^2+8x-48}{x^2+x-12} \equiv x^2+5+\dfrac{3(x+4)}{(x+4)(x-3)}$	M1
	$\equiv x^2+5+\dfrac{3}{(x-3)}$	A1
		(4)

Notes

M1: Divides $x^4+x^3-7x^2+8x-48$ by x^2+x-12 to obtain a quadratic quotient and a linear remainder $\alpha x+\beta$ with α and β non-zero.

A1: Correct quotient x^2+5 and correct remainder $3x+12$.

M1: Writes their answer as $\dfrac{x^4+x^3-7x^2+8x-48}{x^2+x-12} = \text{Their Quotient} + \dfrac{\text{Their Remainder}}{(x+4)(x-3)}$

This may be implied by a correct expression following a correct quotient and remainder or by correct values for A and B as long as there is no incorrect work.

A1: Correct expression $x^2+5+\dfrac{3}{(x-3)}$ or states $A=5$, $B=3$

If they obtain $x^2+5+\dfrac{3}{(x+4)}$ this scores A0, even if they subsequently state $A=5$, $B=3$

	(a) Alternative 1 comparing coefficients/substitution	
	$x^4+x^3-7x^2+8x-48 \equiv (x^2+A)(x^2+x-12)+B(x+4)$ or $x^4+x^3-7x^2+8x-48 \equiv x^2(x^2+x-12)+A(x^2+x-12)+B(x+4)$	M1
	2 correct equations from comparing coefficients or substituting values: e.g. $x^2: A-12=-7$, $x: A+B=8$, const: $-12A+4B=-48$ or e.g. $x=3 \Rightarrow 21=7B$, $x=0 \Rightarrow -48=-12A+4B$	A1
	$\Rightarrow A=\ldots$ or $B=\ldots$	M1
	$A=5, B=3$	A1

Notes

M1: Multiplies up by x^2+x-12 or e.g. $(x-3)(x+4)$ to obtain a correct lhs and one of $(x^2+A)(x^2+x-12)$ or $B(x+4)$ on the rhs.

If $(x^2+A)(x^2+x-12)$ is expanded must see both $x^2(x^2+x-12)$ and $A(x^2+x-12)$

A1: Compares 2 coefficients (or may substitute 2 values) and obtains 2 correct equations.

M1: Solves to obtain one of A or B.

A1: Both values correct.

	(a) Alternative 2 repeated division I	
	$\left(x^4+x^3-7x^2+8x-48\right)\div(x-3)\rightarrow Q_1=x^3+4x^2+5x+23,\ R_1=21$ then $\left(x^3+4x^2+5x+23\right)\div(x+4)\rightarrow Q_2=x^2+5,\ R_2=3$	M1A1
	$\left(x^4+x^3-7x^2+8x-48\right)\div(x+4)(x-3)=x^2+5+\dfrac{21}{(x+4)(x-3)}+\dfrac{3}{x+4}$	M1
	$=x^2+5+\dfrac{21+3x-9}{(x+4)(x-3)}=x^2+5+\dfrac{3(x+4)}{(x+4)(x-3)}=x^2+5+\dfrac{3}{(x-3)}$	A1

Notes

M1: Divides by $(x-3)$ first to give a cubic quotient Q_1 and constant remainder R_1.
Then divides Q_1 by $(x+4)$ to give a quadratic quotient Q_2 and constant remainder R_2.

A1: For $Q_1=x^3+4x^2+5x+23,\ R_1=21,\ Q_2=x^2+5,\ R_2=3$

M1: Writes g(x) as $\mathrm{Q}_2+\dfrac{R_1}{(x+4)(x-3)}+\dfrac{R_2}{x+4}$

A1: Correct expression or correct values for A and B.

	(a) Alternative 3 repeated division II	
	$\left(x^4+x^3-7x^2+8x-48\right)\div(x+4)\rightarrow Q_1=x^3-3x^2+5x-12,\ R_1=0$ then $\left(x^3-3x^2+5x-12\right)\div(x-3)\rightarrow Q_2=x^2+5,\ R_2=3$	M1A1
	$\left(x^4+x^3-7x^2+8x-48\right)\div(x+4)(x-3)=x^2+5+\dfrac{3}{x-3}$	M1A1

Notes

M1: Divides by $(x+4)$ first to give a cubic quotient Q_1 and constant remainder R_1.
Then divides Q_1 by $(x-3)$ to give a quadratic quotient Q_2 and constant remainder R_2.

A1: For $Q_1=x^3-3x^2+5x-12,\ R_1=0,\ Q_2=x^2+5,\ R_2=3$

M1: Writes g(x) as $\mathrm{Q}_2+\dfrac{R_1}{(x+4)(x-3)}+\dfrac{R_2}{x-3}$

A1: Correct expression or correct values for A and B.

	(a) Alternative 4 substituting values	
	$\dfrac{x^4+x^3-7x^2+8x-48}{x^2+x-12}\equiv x^2+A+\dfrac{B}{(x-3)}$ e.g. $x=0\Rightarrow 4=A-\dfrac{B}{3}:\ x=1\Rightarrow\dfrac{45}{10}=1+A-\dfrac{B}{2}$	M1A1
	$4=A-\dfrac{B}{3}:\dfrac{45}{10}=1+A-\dfrac{B}{2}\Rightarrow A=\ldots$ or $B=\ldots$	M1
	$A=5,\ B=3$	A1

Notes

M1: Substitutes 2 values for x
A1: Any 2 correct equations.
M1: Solves to obtain one of A or B.
A1: Correct values for A and B.

(b)	$g'(x) = 2x - \dfrac{3}{(x-3)^2}$ **or** $g'(x) = \dfrac{\left(x^2+x-12\right)\left(4x^3+3x^2-14x+8\right)-\left(x^4+x^3-7x^2+8x-48\right)(2x+1)}{\left(x^2+x-12\right)^2}$	M1A1ft
	$x=4 \Rightarrow g'(4) = 2\times 4 - \dfrac{3}{(4-3)^2} = 5$	M1
	$x=4 \Rightarrow g(4) = 4^2+5+\dfrac{3}{4-3}(=24)$ $\Rightarrow y-24=5(x-4)$ **or e.g.** $y=5x+c \Rightarrow 24=5\times 4+c \Rightarrow c=...$	**d**M1
	$y=5x+4$	A1
		(5)
		(9 marks)

Notes

Note that candidates should not be using a calculator in this part so if there is no algebraic differentiation, they will generally score no marks.

M1: Differentiates their $x^2 + A + \dfrac{B}{x-3}$ to obtain $2x \pm \dfrac{B}{(x-3)^2}$ oe e.g. $2x \pm B(x-3)^{-2}$

or differentiates $\dfrac{x^4+x^3-7x^2+8x-48}{x^2+x-12}$ to obtain an expression of the form

$$g'(x) = \frac{\left(x^2+x-12\right)\times \text{a cubic polynomial} - \left(x^4+x^3-7x^2+8x-48\right)\times \text{a 2 term linear expression}}{\left(x^2+x-12\right)^2}$$

A1ft: $x^2 + A + \dfrac{B}{x-3} \rightarrow 2x - \dfrac{B}{(x-3)^2}$. Follow through on their B or allow the letter B or a made up B.

or $g'(x) = \dfrac{\left(x^2+x-12\right)\left(4x^3+3x^2-14x+8\right)-\left(x^4+x^3-7x^2+8x-48\right)(2x+1)}{\left(x^2+x-12\right)^2}$

which must be correct (nothing to follow through).

M1: Subtitutes $x=4$ into their $g'(x)$ in an attempt to find a numerical gradient.
May be implied by their $g'(4)$ following an attempt to differentiate.

dM1: For the correct method of finding an equation of the tangent with

- $x = 4$
- y an attempt at g(4) which may be implied by their value or by 24
- the gradient an attempt at $g'(4)$ following an attempt to differentiate
- the values correctly placed

If $y = mx + c$ is used they must proceed to a value for c.
Depends on the previous method mark.

A1: $y = 5x + 4$ cao and **cso**
Note that an incorrect value for A in part (a) will give the correct answer in (b) but this mark is cso and should be withheld in such cases.