1. Find

(i) (a) $\int (2x + 3)^{12} \mathrm{d}x$

(2)

(b) $\int \frac{5x}{4x^2 + 1} \mathrm{d}x$

(2)

(ii) Given that

$$\int_a^{2a} \frac{t+1}{t} \mathrm{d}t = \ln 7 \qquad a > 0$$

find the exact value of constant a.

(*Solutions relying on calculator technology are not acceptable.*)

(4)

(Total for Question 1 is 8 marks)

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2. $$y = (2x^2 - 3)\tan\left(\frac{1}{2}x\right) \qquad 0 < x < \pi$$

Given that $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ when $x = \alpha$

(a) show that

$$2\alpha^2 - 3 + 4\alpha \sin \alpha = 0$$

(6)

The iterative formula

$$x_{n+1} = \frac{3}{(2x_n + 4\sin x_n)}$$

can be used to find an approximation for α.

(b) Taking $x_1 = 0.7$, find the values of x_2 and x_3, giving each answer to 4 decimal places.

(2)

(c) By choosing a suitable interval and a suitable function that should be stated, show that $\alpha = 0.7283$ to 4 decimal places.

(2)

(Total for Question 2 is 10 marks)

3. $$g(x) = \frac{x^4 + x^3 - 7x^2 + 8x - 48}{x^2 + x - 12} \qquad x > 3 \qquad x \in \mathbb{R}$$

(a) Given that

$$\frac{x^4 + x^3 - 7x^2 + 8x - 48}{x^2 + x - 12} \equiv x^2 + A + \frac{B}{x - 3}$$

find the values of the constants A and B.

(4)

(b) Hence, or otherwise, find the equation of the tangent to the curve with equation $y = g(x)$ at the point where $x = 4$. Give your answer in the form $y = mx + c$, where m and c are constants to be determined.

(*Solutions relying on calculator technology are not acceptable.*)

(5)

(Total for Question 3 is 9 marks)

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4. A curve has equation

$$y = \ln(1 - \cos 2x) \qquad x \in \mathbb{R} \qquad 0 < x < \pi$$

Show that

(a) $\frac{dy}{dx} = k \cot x$, where k is a constant to be found.

(4)

Hence find the exact coordinates of the point on the curve where

(b) $\frac{dy}{dx} = 2\sqrt{3}$

(4)

(Total for Question 4 is 8 marks)