(ii)	$\int \frac{t+1}{t}\,\mathrm{d}t = \int\left(1+\frac{1}{t}\right)\mathrm{d}t = t+\ln t\,(+c)$	M1 A1
	$\left[t+\ln t\right]_a^{2a} = 2a+\ln 2a-(a+\ln a) = \ln 7$	**d**M1
	$a+\ln 2 = \ln 7 \Rightarrow a = \ln\frac{7}{2}$ or $\ln 7-\ln 2$	A1
		(4)
		(8 marks)

(ii) Notes

M1: Writes $\frac{t+1}{t}$ as $...+\frac{1}{t}$ where … $\neq 0$ and integrates $\frac{1}{t} \to \ln t$

A1: Correct integration with or without an arbitrary constant.

dM1: Applies the limits a and $2a$ the right way round, subtracts and sets equal to $\ln 7$

Condone poor bracketing e.g. $2a+\ln 2a-a+\ln a = \ln 7$

Depends on the first method mark.

A1: $a = \ln\frac{7}{2}$ or e.g. $\ln 3.5$

Condone $a = \ln 7-\ln 2$ but if this becomes $\ln 5$ score A0

Condone clear confusion with the variable if the intention is clear e.g.

$$\int_a^{2a}\frac{t+1}{t}\,\mathrm{d}t = \int_a^{2a}\left(1+\frac{1}{t}\right)\mathrm{d}t = \left[x+\ln t\right]_a^{2a} \to 2a+\ln 2a-(a+\ln a) = \ln 7 \text{ etc.}$$

Alternative attempts by parts for the first 2 marks:

I	$\int \frac{t+1}{t}\,\mathrm{d}t = \frac{1}{t}\left(\frac{t^2}{2}+t\right)-\int -\frac{1}{t^2}\left(\frac{t^2}{2}+t\right)\mathrm{d}t$	M1
	$= \frac{1}{t}\left(\frac{t^2}{2}+t\right)+\int\frac{1}{2}+\frac{1}{t}\,\mathrm{d}t = \frac{1}{t}\left(\frac{t^2}{2}+t\right)+\frac{t}{2}+\ln t\,(+c)$	A1

Notes

M1: Award for $\int \frac{t+1}{t}\,\mathrm{d}t = \frac{1}{t}\left(\frac{t^2}{2}+t\right)-\int -\frac{1}{t^2}\left(\frac{t^2}{2}+t\right)\mathrm{d}t$

Must be correct as shown.

A1: Fully correct integration. $\frac{1}{t}\left(\frac{t^2}{2}+t\right)+\frac{t}{2}+\ln t\,(+c)$ or equivalent.

II	$\int \frac{t+1}{t}\,\mathrm{d}t = (t+1)\ln t-\int \ln t\,\mathrm{d}t$	M1
	$\int \frac{t+1}{t}\,\mathrm{d}t = (t+1)\ln t-t\ln t+t\,(+c)$	A1

Notes

M1: Award for $\int \frac{t+1}{t}\,\mathrm{d}t = (t+1)\ln t-\int \ln t\,\mathrm{d}t$

Must be correct as shown.

A1: Fully correct integration. $(t+1)\ln t-t\ln t+t\,(+c)$ or equivalent.

Question Number	Scheme	Marks
2(a)	$y=(2x^2-3)\tan\left(\frac{1}{2}x\right)\Rightarrow\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)4x\tan\left(\frac{1}{2}x\right)+(2x^2-3)\times\frac{1}{2}\sec^2\left(\frac{1}{2}x\right)$	M1A1A1
	$x=\alpha\Rightarrow 4\alpha\tan\left(\frac{1}{2}\alpha\right)+(2\alpha^2-3)\times\frac{1}{2}\sec^2\left(\frac{1}{2}\alpha\right)=0$ $\Rightarrow 8\alpha\frac{\sin\left(\frac{1}{2}\alpha\right)}{\cos\left(\frac{1}{2}\alpha\right)}+(2\alpha^2-3)\times\frac{1}{\cos^2\left(\frac{1}{2}\alpha\right)}=0$	M1
	$8\alpha\sin\left(\frac{1}{2}\alpha\right)\cos\left(\frac{1}{2}\alpha\right)+(2\alpha^2-3)=0$ $4\alpha\sin\alpha+(2\alpha^2-3)=0$	**d**M1
	$2\alpha^2-3+4\alpha\sin\alpha=0*$	A1*
		(6)

Notes

Brackets around the $\frac{1}{2}x$ are not required.

M1: Attempts the product rule on $y=(2x^2-3)\tan\left(\frac{1}{2}x\right)$ or on $y=2x^2\tan\left(\frac{1}{2}x\right)$ if they multiply out.
It is for **either** $(2x^2-3)\tan\left(\frac{1}{2}x\right)\to Ax\tan\left(\frac{1}{2}x\right)+B(2x^2-3)\sec^2\left(\frac{1}{2}x\right),\ A,B>0$
or for $2x^2\tan\left(\frac{1}{2}x\right)\to Ax\tan\left(\frac{1}{2}x\right)+Bx^2\sec^2\left(\frac{1}{2}x\right),\ A,B>0$

A1: One term correct: of $4x\tan\left(\frac{1}{2}x\right)$ or $(2x^2-3)\times\frac{1}{2}\sec^2\left(\frac{1}{2}x\right)$ or $(2x^2)\times\frac{1}{2}\sec^2\left(\frac{1}{2}x\right)$

A1: e.g. $\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)4x\tan\left(\frac{1}{2}x\right)+(2x^2-3)\times\frac{1}{2}\sec^2\left(\frac{1}{2}x\right)$.

A fully correct unsimplified or simplified derivative. $\frac{\mathrm{d}y}{\mathrm{d}x}=$ is not required.

M1: They need to have a derivative with both $\tan\left(\frac{1}{2}x\right)$ and $\sec^2\left(\frac{1}{2}x\right)$. It is for using

$\tan\left(\frac{1}{2}x\right)=\frac{\sin\left(\frac{1}{2}x\right)}{\cos\left(\frac{1}{2}x\right)}$ and $\sec^2\left(\frac{1}{2}x\right)=\frac{1}{\cos^2\left(\frac{1}{2}x\right)}$ or $\sec^2\left(\frac{1}{2}x\right)=1+\tan^2\left(\frac{1}{2}x\right)=1+\frac{\sin^2\left(\frac{1}{2}x\right)}{\cos^2\left(\frac{1}{2}x\right)}$

and setting $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ which may be implied. This mark is for converting to an equation in $\sin\left(\frac{1}{2}x\right)$ and $\cos\left(\frac{1}{2}x\right)$ only using the correct identities which may be implied by their work.

dM1: Uses the correct identity $2\sin\left(\frac{1}{2}x\right)\cos\left(\frac{1}{2}x\right)=\sin x$.
This may be implied by their work but if the identity $\sin\left(\frac{1}{2}x\right)\cos\left(\frac{1}{2}x\right)=\sin x$ is suggested, score dM0.
Depends on the previous method mark.

A1*: $2\alpha^2-3+4\alpha\sin\alpha=0$ (Allow $4\alpha\sin\alpha+2\alpha^2-3=0$).
This is a printed answer so must follow all previous marks.
There must have been no errors, including bracketing errors.
May work in x or α (or a) or a mixture of both but must be α or a for final A1

(b)	$x_2 = \dfrac{3}{\left(2\times 0.7 + 4\sin 0.7\right)}$	M1
	$x_2 = 0.7544, x_3 = 0.7062$	A1
		(2)

Notes

(b)

M1: For substituting $x_1 = 0.7$, into the given iterative formula to find a value for x_2.

It may be implied by the sight of $x_2 = \dfrac{3}{\left(2\times 0.7 + 4\sin 0.7\right)} = ...$ or $x_2 =$ awrt 0.75 and also (if degrees were used) $x_2 =$ awrt 2.1

Do not condone misreads of the given iteration formula.

A1: $x_2 = 0.7544, x_3 = 0.7062$ (both awrt 4 dp)

Remember to isw so allow e.g. $x_2 = 0.75436 = 0.7543, x_3 = 0.7062$

(c)	Chooses interval e.g. $[0.72825, 0.72835]$ AND function, e.g. $f(x) = 2x^2 - 3 + 4x\sin x$ and attempts the value of $f(0.72825)$ and $f(0.72835)$	M1
	$2\times0.72825^2 - 3 + 4\times0.72825\sin 0.72825 = -0.0005$ $2\times0.72835^2 - 3 + 4\times0.72835\sin 0.72835 = 0.00026$ + reason + conclusion	A1
		(2)
		(10 marks)

Notes

(c)

M1: Chooses

- A suitable interval e.g. $[0.72825, 0.72835]$
- A suitable function and attempts to find the value of the function at both ends.

The function may be implied by embedded values e.g.

$2(0.72825)^2 - 3 + 4(0.72825)\sin(0.72825)$ and $2(0.72835)^2 - 3 + 4(0.72835)\sin(0.72835)$

or by their working/values e.g. $0.72825 \rightarrow \frac{dy}{dx} = \ldots$, $0.72835 \rightarrow \frac{dy}{dx} = \ldots$,

Accept as suitable functions, $\pm(2\alpha^2 - 3 + 4\alpha\sin\alpha)$, $\pm\left(x - \frac{3}{(2x + 4\sin x)}\right)$, their $\frac{dy}{dx}$

Allow a smaller interval containing the root.

The interval does not have to be stated so the values may be seen at the point of substitution.

Condone the use of degree mode – see table below for values.

A1: Achieves

- Correct values to 1 sf rounded or truncated
- Gives a valid reason e.g. change of sign (or equivalent e.g. > 0, < 0)
- Minimal conclusion such as "hence root", "turning point", "proven"

There is no requirement to reference continuity and any mention of it can be ignored.

For reference:

Function f	f (0.72825)	f (0.72835)
$2x^2 - 3 + 4x\sin x$	–0.00051431… (Degrees = –1.90227…)	0.00026066… (Degrees = –1.90197…)
$x - \frac{3}{(2x + 4\sin x)}$	–0.00012487… (Degrees = –1.26201…)	0.000063278… (Degrees = –1.26163…)
$\frac{dy}{dx}$	–0.0002945… (Degrees = –0.95117…)	0.000149267 (Degrees = –0.95102…)

Attempts at repeated iteration score no marks in (c).