1. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The functions f and g are defined by

$$f(x) = \ln(x^2 + 3) \qquad x \in \mathbb{R}$$

$$g(x) = \frac{3 + 5x}{x + 2} \qquad x \in \mathbb{R} \quad x > -2$$

(a) State the range of f

(1)

(b) Find g^{-1}

(3)

(c) Find $fg(0)$

(2)

(d) Find the exact value of a for which

$$g\left(e^{2a}\right) = f\left(\sqrt{e^4 - 3}\right)$$

(3)

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Question 1 continued

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Question 1 continued

(Total for Question 1 is 9 marks)

2.

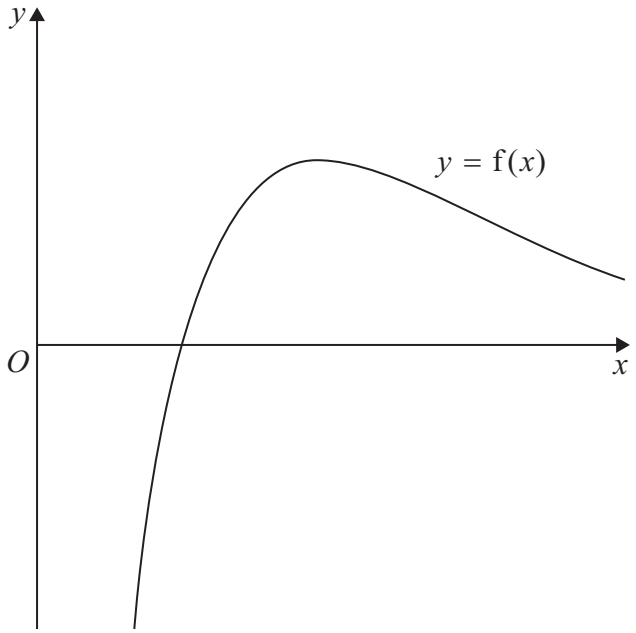


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = \frac{2x^2 + 3x - 4}{\mathrm{e}^x} - \frac{1}{x^2} \qquad x \in \mathbb{R} \quad x \neq 0$$

(a) Show that $\mathrm{f}(x) = 0$ has a root α in the interval $[1, 2]$

(2)

(b) Show that the equation $\mathrm{f}(x) = 0$ can be written in the form

$$x = \sqrt[3]{\frac{\mathrm{e}^x + 4x^2}{2x + 3}}$$

(2)

Using the iteration formula

$$x_{n+1} = \sqrt[3]{\frac{\mathrm{e}^{x_n} + 4x_n^{\,2}}{2x_n + 3}} \qquad \text{with } x_1 = 1$$

find, to 4 decimal places,

(c) (i) the value of x_3

(ii) the value of α

(3)

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