Question Number	Scheme	Marks
7 (a)	$\sin 4\theta \equiv 2\sin 2\theta\cos 2\theta$	M1
	$\equiv 2\left(2\sin\theta\cos\theta\right)\left(1-2\sin^2\theta\right)$	dM1
	$\equiv 4\sin\theta\cos\theta - 8\sin^3\theta\cos\theta$	
	$\equiv \sin\theta\cos\theta\left(4-8\sin^2\theta\right)$	A1
		(3)
(b)	$\sec x\sin 4x = 5\sin^3 x\cot x$	
	$\frac{1}{\cos x}\times\cos x\sin x\left(4-8\sin^2 x\right) = 5\sin^3 x\cot x$	B1
	$\div\sin x \Rightarrow\ 4-8\sin^2 x = 5\sin^2 x\cot x$	
	$\div\cos^2 x \Rightarrow\ 4\sec^2 x - 8\tan^2 x = 5\tan^2 x\cot x$	M1
	$\Rightarrow 4\sec^2 x - 5\tan x - 8\tan^2 x = 0$ *	A1*
		(3)
(c)	Uses $\sec^2 x = 1+\tan^2 x$	M1
	$\Rightarrow 4\tan^2 x + 5\tan x - 4 = 0$	A1
	$\Rightarrow \tan x = \frac{-5\pm\sqrt{89}}{8} \Rightarrow x = ...$	dM1
	$\Rightarrow x =$ awrt 0.506, 2.08	A1
		(4)
		(10 marks)

(a)

M1: Uses the double angle formula for $\sin 4\theta$ to get $2\sin 2\theta\cos 2\theta$ o.e. such as $\sin 2\theta\cos 2\theta + \cos 2\theta\sin 2\theta$

dM1: Uses the double angle formulae $\sin 2\theta = K\sin\theta\cos\theta$ and $\cos 2\theta = \pm 1\pm 2\sin^2\theta$ to obtain $\sin 4\theta$ in terms of just $\sin\theta$ and $\cos\theta$. If they use one of the other identities for $\cos 2\theta$, either $\pm 2\cos^2\theta \pm 1$ or $\cos^2\theta - \sin^2\theta$, they must subsequently change the $\cos^2\theta$ terms to $1-\sin^2\theta$

A1: Reaches $\sin 4\theta \equiv \sin\theta\cos\theta\left(4-8\sin^2\theta\right)$

Withold this mark if there are mixed variables e.g. $\sin 4\theta \equiv 2\sin 2x\cos 2x$ and incorrect notation e.g. $\cos 2\theta \equiv 1-2\sin\theta^2$ used within the body of the proof.

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You may well see more complicated versions of this proof.

For example,

$\sin 4\theta \equiv \sin\left(3\theta+\theta\right) = \sin 3\theta\cos\theta + \cos 3\theta\sin\theta$

followed by $= \sin\left(2\theta+\theta\right)\cos\theta + \cos\left(2\theta+\theta\right)\sin\theta$

$= \sin 2\theta\cos^2\theta + \cos 2\theta\sin\theta\cos\theta + \cos 2\theta\cos\theta\sin\theta - \sin 2\theta\sin\theta\sin\theta$

In this method the M1 is scored for the line above but you can condone sign slips

And the dM1 for using the double angle formulae $\sin 2\theta = 2\sin\theta\cos\theta$ and $\cos 2\theta = \pm 1 \pm 2\sin^2\theta$ to obtain $\sin 4\theta$ in terms of just $\sin\theta$ and $\cos\theta$

Do not allow candidates just to write out the expansions for $\sin 3\theta$ and $\cos 3\theta$ without their proof

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(b)

B1: States or uses $\sec x = \dfrac{1}{\cos x}$ in the given equation.

It can be implied. So, writing $\sec x \sin 4x$ as $\sin x\left(4 - 8\sin^2 x\right)$ would be B1

M1: Requires

- $\sin 4x$ to be written as $\sin x \cos x\left('4' - '8'\sin^{'2'} x\right)$ using their values for P, Q and n
- a term in $\sin x$ to be cancelled or factorised out
- an attempt at dividing all terms by $\cos^2 x$ to reach an equation involving at least $\sec^2 x$ and $\tan^2 x$ (you don't need to see $\div\cos^2 x$ stated, it can be implied by their terms)

The last two bullets can be done in just one line and implied by all terms $\div\left(\sin x \cos^2 x\right)$

A1*: Reaches the given equation showing sufficient and necessary lines of work.

Withold this mark for mixed variables and/or incorrect notation within the proof.

(c)

M1: Uses $\sec^2 x = \pm 1 \pm \tan^2 x$ to set up a 3TQ in $\tan x$

A1: Correct 3TQ, $4\tan^2 x + 5\tan x - 4 = 0$.

The terms do not need to be on one side and the = 0 can be implied by further work

dM1: Solves the 3TQ using an appropriate method and proceeds to a value for x via value(s) for $\tan x$

Expect to see $"4"\tan^2 x + "5"\tan x - "4" = 0 \Rightarrow \tan x = ... \Rightarrow x = ...$

Allow calculator methods of solving the quadratic in $\tan x$ to produce a value for $\tan x$

A1: $x =$ awrt 0.506, 2.08 following M1, A1, dM1.

Ignore any extra solutions outside the range $(0, \pi)$.

Withold this mark if there are extra solutions in the range.

Note that the demand of the question is that candidates show their working.

So, from the given $4\sec^2 x - 5\tan x - 8\tan^2 x = 0$

candidates writing $\Rightarrow 4\tan^2 x + 5\tan x - 4 = 0$ followed by $x =$ awrt 0.506, 2.08 scores just the first M1, A1

There will be alternatives seen in part (c).

M1: Equation in a single trig ratio A1: Correct equation dM1: Full attempt to solve A1: Correct solutions

Alt (c)

$$4\sec^2 x - 5\tan x - 8\tan^2 x = 0$$

$$\times\cos^2 x \Rightarrow 4 - 5\sin x\cos x - 8\sin^2 x = 0$$

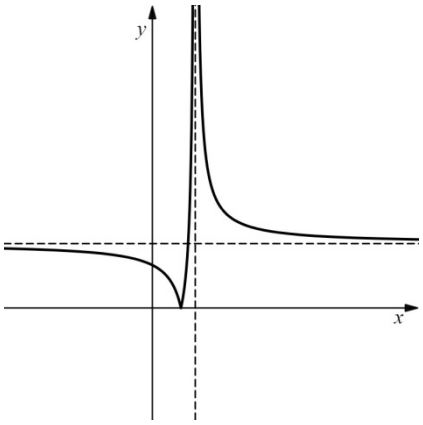
$$\Rightarrow 4 - 8\sin^2 x - 5\sin x\cos x = 0$$

$$\Rightarrow 4\cos 2x - 2.5\sin 2x = 0$$

$$\Rightarrow \tan 2x = \frac{8}{5} \qquad \text{M1, A1}$$

$$\Rightarrow 2x = 1.0121, 4.1538$$

$$\Rightarrow x = \text{awrt } 0.506, 2.08 \qquad \text{dM1, A1}$$

Question Number	Scheme	Marks
8. (a)	$g(2a) = \dfrac{3\times 2a - 2a}{2a - a} = 4$, $\quad f(4) = \ln(2\times 4 - 3) = \ln 5$	M1, A1 **(2)**
(b)	$y = \ln(2x-3) \Rightarrow e^{y} = 2x - 3 \Rightarrow x = \dfrac{e^{y}+3}{2}$	M1
	$f^{-1}(x) = \dfrac{e^{x}+3}{2}$ $\qquad x > 0$	A1, B1 **(3)**
(c)	 Shape of curve to right of $x = a$ Shape of curve to left of $x = a$ $y = 3$ marked at correct place on curve	B1 B1 B1 **(3)**
(d) (i)	$\dfrac{3\times 8 - 2a}{8 - a} = 10 \Rightarrow a = ...$	M1
	$\Rightarrow a = 7$	A1
(ii)	$\dfrac{3x - 2\times 7}{x - 7} = -10 \Rightarrow x = ...$	dM1
	$\Rightarrow x = \dfrac{84}{13}$	A1 **(4)**
		(12 marks)

(a)

M1: Complete method to find fg($2a$). E.g. Finds g($2a$) and substitutes into f.

Some may find fg(x) = $\ln\left(2\times\dfrac{3x-2a}{x-a} - 3\right)$ and substitute $x = 2a$ into this. Condone slips in signs

A1: $\ln 5$. ISW after a correct answer. Allow this to be written down for both marks.