Question Number	Scheme	Marks
3. (a)	$\dfrac{\mathrm{d}x}{\mathrm{d}y}=\dfrac{8y(2y+1)-2\left(4y^2-3\right)}{(2y+1)^2}$	M1 A1
	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{(2y+1)^2}{8y^2+8y+6}$	dM1, A1
		(4)
(b)	Sets $\dfrac{(2y+1)^2}{8y^2+8y+6}=\dfrac{1}{3}\Rightarrow 4y^2+4y-3=0$	M1, A1
	$\Rightarrow (2y-1)(2y+3)=0\Rightarrow y=\dfrac{1}{2}, x=\ldots$	dM1
	$P=\left(-1,\ \dfrac{1}{2}\right)$ only	A1
		(4) **(8 marks)**

(a)

M1: Attempts the quotient rule to achieve $\dfrac{Ay(2y+1)-B\left(4y^2-3\right)}{(2y+1)^2}$ $A>0, B>0$.

Product rule attempts may be seen

Condone missing brackets and ignore the LHS, so $\dfrac{\mathrm{d}x}{\mathrm{d}y}$ may be $\dfrac{\mathrm{d}y}{\mathrm{d}x}$

A1: $\dfrac{\mathrm{d}x}{\mathrm{d}y}=\dfrac{8y(2y+1)-2\left(4y^2-3\right)}{(2y+1)^2}$ with both left-hand and right-hand sides correct (unsimplified)

Via the product rule you should see $\dfrac{\mathrm{d}x}{\mathrm{d}y}=8y(2y+1)^{-1}-2\left(4y^2-3\right)(2y+1)^{-2}$

dM1: Uses $\dfrac{\mathrm{d}y}{\mathrm{d}x}=1\div\dfrac{\mathrm{d}x}{\mathrm{d}y}$. It is dependent upon the previous M1

A1: $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{(2y+1)^2}{8y^2+8y+6}$ o.e such as $\dfrac{4y^2+4y+1}{8y^2+8y+6}$ ISW after a correct answer.

Allow partially factorised forms such as $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{(2y+1)^2}{2(2y+1)^2+4}$ but $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{-4y^2-4y-1}{-8y^2-8y-6}$ is A0

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…….

Alt via division

M1: Divides $\left(4y^2-3\right)$ by $(2y+1)$ to achieve $2y+\alpha+\dfrac{\beta}{2y+1}$ and then differentiates to

$2+\dfrac{\delta}{(2y+1)^2}$

A1: Divides $\left(4y^2-3\right)$ by $\left(2y+1\right)$ to achieve $2y-1-\dfrac{2}{2y+1}$ and then differentiates to

$\dfrac{\mathrm{d}x}{\mathrm{d}y}=2+\dfrac{4}{\left(2y+1\right)^2}$

dM1: Requires $\dfrac{\mathrm{d}x}{\mathrm{d}y}=2+\dfrac{\delta}{\left(2y+1\right)^2}$ with attempt to find the reciprocal to reach $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{\left(2y+1\right)^2}{2\left(2y+1\right)^2+\delta}$

A1: Simplifies to $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{\left(2y+1\right)^2}{8y^2+8y+6}$ or $\dfrac{4y^2+4y+1}{8y^2+8y+6}$

……

(b)

M1: Sets their $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\text{"}\dfrac{\left(2y+1\right)^2}{8y^2+8y+6}\text{"}=\dfrac{1}{3}$ and proceeds to a simplified quadratic equation in y.

Alternatively sets their $\dfrac{\mathrm{d}x}{\mathrm{d}y}=\dfrac{8y^2+8y+6}{\left(2y+1\right)^2}=3$ and proceeds to a simplified quadratic equation in y.

A1: Correct 3TQ E.g. $4y^2+4y-3=0$ but allow variations such as $4y^2=3-4y$

dM1: Solves quadratic via a correct method, finds a positive y value and the associated x coordinate.

Allow the quadratic to be solved via a calculator but it must be the simplified 3TQ

It is dependent upon the previous M mark.

This can be awarded if they find both values and cross out the correct answer.

A1: $P=\left(-1,\dfrac{1}{2}\right)$ only. Allow $x=-1,\ y=\dfrac{1}{2}$ ISW after a correct answer

If they find both values $\left(-1,\dfrac{1}{2}\right)$ and $\left(-3,-\dfrac{3}{2}\right)$ AND don't reject $\left(-3,-\dfrac{3}{2}\right)$ they will lose the last mark only

The demand is that solutions relying on calculator technology are not acceptable.

If a candidate uses their calculator to solve a correct $\dfrac{\left(2y+1\right)^2}{8y^2+8y+6}=\dfrac{1}{3}$ o.e they can be awarded M1 for

$y=\dfrac{1}{2}$ and A1 for $x=-1,\ y=\dfrac{1}{2}$ for a total of 1100 in (b)

……..

For candidates who have studied WMA14 you may see versions of implicit differentiation

E.g $2xy+x=4y^2-3\Rightarrow 2y+2x\dfrac{\mathrm{d}y}{\mathrm{d}x}+1=8y\dfrac{\mathrm{d}y}{\mathrm{d}x}$

$\Rightarrow\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{2y+1}{8y-2x}$

Then substitute $x=$ into $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{2y+1}{8y-2x}\Rightarrow\dfrac{2y+1}{8y-2\times\dfrac{4y^2-3}{2y+1}}=\dfrac{\left(2y+1\right)^2}{8y^2+8y+6}$

Score M1 for an attempt at both product and chain rule on xy and y^2 terms

A1 for a correct $\frac{dy}{dx}$ in terms of both x and y

dM1 for the substitution and attempt at simplification

A1 Fully correct and simplified

Note that part (b) can be attempted from $\frac{2y+1}{8y-2x}=\frac{1}{3} \Rightarrow x=y-\frac{3}{2}$ o.e which can then be substituted into the equation of the curve.

Question Number	Scheme	Marks
4.(a)	$x^2+0x+1\overline{)3x^3-5x^2+7x-5}$ $\quad 3x-5$ $3x^3+0x^2+3x$ $-5x^2+4x$ $-5x^2+0x-5$ $4x$	M1
	$\underline{A=3}, \quad B=-5, C=4$	<u>B1</u>, A1
	$D=0$ *	A1*
		(4)
(b)	$\int 3x-5+\frac{4x}{x^2+1}\,\mathrm{d}x=\frac{3x^2}{2}-5x+2\ln(x^2+1)+c$	M1, A1ft
	$\text{Area}=\left(\frac{3\times3^2}{2}-5\times3+2\ln(3^2+1)\right)-\left(\frac{3\times2^2}{2}-5\times2+2\ln(2^2+1)\right)$	
	$=\frac{5}{2}+\ln 4$	dM1, A1
		(4) **(8 marks)**

(a) **Method via division**

M1: Full attempt to divide producing a linear quotient $(...x+...)$ and linear remainder $(...x+...)$

B1: States $A = 3$ which may be awarded from the quotient line in the division sum $3x...$

This is an independent mark and can be awarded without sight of method

A1: $B = -5$ and $C = 4$ following the award of the M mark. May be awarded from within the division sum.

A1*: Requires

- fully correct division (see main scheme) with no constant within the remainder which proves that $D = 0$
- all previous marks to have been scored
- all constants stated or correct expression seen in (a) or (b)

If the candidates don't put in the $'0'x$ you would need to see something like the following work

$x^2+1\overline{)3x^3-5x^2+7x-5}$ $\quad 3x-5$

$3x^3\quad\quad+3x$

$-5x^2+4x-5$

$-5x^2\quad\quad-5$

$4x$

The remainder should be $4x$ and not $4x \pm 5$